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CONTENTS

INVITED SPEAKERS

Optimization problems with oscillating controls

Bronisław Jakubczyk, *Warsaw, Poland*

We will discuss optimal control problems related to infinitesimally optimal path following. This leads to nilpotent approximations and optimal problems on nilpotent Lie groups. Based on a common work with J.-P. Gauthier and V. Zakalyukin, we will show how harmonic oscillatory controls solve the problem for 1-step non-holonomic systems. We will give an explicit formula for a measure of complexity of the problem, called the entropy. For more general systems one should solve certain open optimization problems involving Lie polynomials.

Stochastic Controllability of Fractional Linear Systems

Jerzy Klamka, *Gliwice, Poland*

Controllability plays an important role both in deterministic and stochastic control theory. In the literature there are many different definitions of controllability, both for linear and nonlinear dynamical systems, which strongly depend on class of dynamical control systems and the set of admissible controls.

However, it should be stressed, that the most literature in this direction has been mainly concerned with deterministic controllability problems for finite-dimensional linear dynamical systems with standard derivative in the differential state equation.

Controllability concepts for stochastic control systems have been recently discussed only in a rather few number of publications.

In the present paper we shall study stochastic controllability problems for fractional linear dynamical systems, which are natural generalizations of controllability concepts well known in the theory of infinite dimensional control systems. It will be proved that under suitable assumptions controllability of a deterministic fractional linear associated dynamical system is equivalent to stochastic exact controllability and stochastic approximate controllability of the original fractional linear stochastic dynamical system. This is a generalization to fractional case some previous results concerning stochastic controllability of linear dynamical systems.

The paper is organized as follows: section 2 contains mathematical model of linear, fractional stationary stochastic dynamical system. In section 3 using results and methods taken directly from deterministic controllability problems, necessary and sufficient conditions for exact and approximate stochastic controllability are formulated and proved. Finally, section 4 contains concluding remarks.

Cell nuclei detection for computerized cancer diagnosis based on stochastic geometry and deep learning

Jozef Korbicz, *Zielona Gora, Poland*
Marek Kowal, *Zielona Gora, Poland*

A modern cancer diagnostic is based heavily on cytological tests. Unfortunately, experienced pathologists need a lot of time to inspect cell nuclei coming from the tissue sample. Such a diagnosis can be facilitated and speeded up by using automatic image segmentation and analysis methods. But, we have to take into account the fact that a cytological image is a hard problem for computer vision because tissue samples are composed of complex cellular structures. Classical segmentation methods such as thresholding, active contours or watershed transform are effective only for simple cases where nuclei are well isolated from each other. In case of cytological material, this requirement is very rarely fulfilled. To tackle this problem a hybrid approach based on convolutional neural network (CNN) and stochastic geometry is proposed for automated detection of nuclei. We can observe that more and more nuclei segmentation and detection approaches are based on CNN. The main advantage of this approach is that CNN learns from training data a hierarchy of filters to extract invariant features to segment images. This approach has proven to be more accurate for semantic segmentation than methods based on features engineered by hand. However, cytological images are specific in this sense that nuclei usually are clumped, and therefore occlusions are very frequent. Consequently, we typically need to detect nuclei which are only partially visible. To overcome this problem, results of semantic segmentation are post-processed with the help of stochastic geometry to extract nuclei from clusters which CNN missed to detect. Nuclei distribution is modeled by the stochastic process, and then Besag's iterated conditional modes approach is applied to find the configuration of nuclei models that fit the input image best. The method is tuned to detect cell nuclei that are partially occluded or create dense clusters. To test the effectiveness of the proposed method, it was applied to detect nuclei in breast cancer cytological images. Detection accuracy was determined concerning reference results obtained by manual segmentation of cell nuclei. The proposed approach has led to better results than the marker-controlled watershed both in the number of correctly detected nuclei and in the number of false detections.

On exact controllability and complete stabilizability of linear systems in Hilbert spaces

Rabah Rabah, *Nantes, France*

We consider linear systems in the general form

$$\dot{x} = \mathcal{A}x + \mathcal{B}u, \tag{1}$$

where the state $x(t)$ and the control $u(t)$ take values in Hilbert spaces X and U. A is a linear operator, infinitesimal generator of a C_0-semigroup $S(t)$, B is linear bounded operator. By exact (null) controllability we mean controllability from any state to any state (or zero state). By complete stabilizability we mean exponential stabilizability with arbitrary decay rate or, sometimes pole assignment, by linear state feedback $u = \mathcal{F}x$.

It is well known (cf. for example [2]) that in an finite dimensional setting exact controllability (said complete controllability) is a necessary and sufficient condition for complete stabilizability or more precisely arbitrary pole assignment. The situation is more complicated in infinite dimensional spaces.

We recall some classical results concerning the relation between exact controllability and complete stabilizability.

The first important result in this context was given by Slemrod [1]: if $S(t)$ is a group, exact controllability implies complete stabilizability. The converse, for a group, was proved by Zabczyk [3]. The result was generalized and precized by several authors for the case of a bounded operator A, for the case of a semigroup $S(t)$ (not a group) and for some classes of systems, governed by partial differential equations or functional-differential equations (with delays).

We discuss more precisely the relations between exact null controllability and complete stabilizability. For the system (1), exact null controllability implies complete stabilizability, but the converse is not true. We give more recent results on functional-differential systems of neutral type, which may be represented in the form of system (1), and described by the equation:

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)\,\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)\,\mathrm{d}\theta + Bu.$$

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Affine varieties in the tangent bundle

Michail Zhitomirskii, *Technion, Israel*

An affine variety in the tangent bundle is a subset of the tangent bundle whose intersection with any tangent space is an affine (in particular vector) subspace, not necessarily of constant dimension, which can be locally described either by vector fields or by differential forms. The local classification of affine varieties is a big tuple of directions of research in local differential geometry, geometric control theory, dynamical systems, and singularity theory. In the talk, the main direction of research, tools, and results will be conceptually characterized.

SHORT COMMUNICATIONS

The KdV equation as a Hamiltonian system. Symplectic form in terms of left scattering data

Kyrylo Andreiev, *Kharkiv, Ukraine*

For the Korteweg-de Vries equation

$$q_t(x,t) - 6q(x,t)q_x(x,t) + q_{xxx}(x,t) = 0$$

with steplike initial profile $q(x,0) = q(x)$, which is of the Schwartz type in the following meaning:

$$\int_{\mathbb{R}_+} x^m(|q(-x) - c^2| + |q(x)|)dx + \int_{\mathbb{R}} |x|^m |q^{(s)}(x)| dx < \infty, \quad \forall m, s \geq 1,$$

we propose a representation of the sympletic form in terms of the left scattering data. Our work generalizes the well known result of V.E. Faddeev and L.D. Zakharov [1].

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One problem of a control system design in the class C^1

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In the paper [1] a method for constructing a controllable system was proposed for the class of real-analytic vector fields. Let $\dot{x} = f(x)$ be a system of differential equations, where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}^n$, $n > 1$. The problem is to find a vector field $g(x)$ such that the system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

is completely controllable. The following theorem was proved in [1]:

Theorem 1. *Let a system $\dot{x} = f(x)$ be given. A vector field $g(x)$, for which the system (1) is controllable, exists if and only if $f \not\equiv 0$.*

The idea [1] of constructing a controllable system in the theorem is to straighten the given vector field $f(x)$, that is, to obtain $f = [0, \dots, 1]^T$ and, changing variables, to obtain a linear system that we already know how to make controllable. By performing the inverse change of variables, we obtain a controllable system in the initial coordinates.

In the report, this problem is considered for finitely differentiable vector fields, in particular, for vector fields of the class C^1. In this case a system design problem is related to linearizability problems studied in the paper [2]. As a result, we obtain the following theorem:

Theorem 2. *Let a system of differential equations*

$$\dot{x} = f(x) + g(x)u$$

be given, where the vector field $g(x)$ is constructed according to the method described above. If the vector field $f(x)$ is of the class C^2, then the vector field $g(x)$ is of the class C^1.

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One method of mapping nonlinear systems to linear

Nataliia Averianova, *Kharkiv, Ukraine*
Aleksandr Svetlichny, *Kharkiv, Ukraine*

We consider the problems of searching for admissible control, such that the given initial point passes to a given finite value at a given time due to a system with a nonlinear right-hand side.

We study the possibility of replacing variables, as proposed for triangular systems, to systems that are not triangular. We consider several cases: systems which are non-linear with respect to the control, two-dimensional systems in which the first equation does not depend on the second coordinate, and systems in which the right-hand side depends only on the control. Various methods were used for constructing the control. For the considered problems, we found the control of two types: in the form of piecewise constant functions and in the form of polynomials.

We considered several examples and found several controls by different methods.

Also, we consider a three-dimensional system, which was studied in connection with the time-optimal problem in the paper by S. Yu. Ignatovich [3]. The applied methods are based on Korobov's theory of triangular systems (the transformation of nonlinear systems into linear ones).

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Exact null controllability of time-delay systems as trigonometric moment problem

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Grigory Sklyar, *Szczecin, Poland*

We analyze the relation between notions of exact null controllability and spectral controllability for a quite general class of linear time-delay systems of retarded type with distributed terms. One of the first results was obtained in [2] where two-dimensional systems of the form

$$\dot{x} = A_1 x(t-h) + A_0 x(t) + bu(t) \tag{1}$$

were considered. The authors proved that exact null controllability is equivalent to spectral controllability for such systems. In 1979 V. Marchenko [3] conjectured that this equivalence holds for much more general class of retarded systems. In 1984 Colonius [1] showed the equivalence property for systems (1) of arbitrary dimensions. His proof was based on the fact that spectrum controllability is equivalent to solvability of finite spectrum assignment problem. Later in [4] an explicit algebraic algorithm of computing a control function which steers any given initial function to the equilibrium position in finite time was given. This allowed to prove that spectral controllability implies null controllability for quite wide class of systems.

In this work we consider a more broad class of systems given by

$$\dot{z}(t) = A_1 z(t-1) + \int_{-1}^{0} [A_2(\theta)\dot{z}(t+\theta) + A_3(\theta)z(t+\theta)]\,\mathrm{d}\theta + Bu(t), \tag{2}$$

assuming that $\mathrm{rank}(A_1,\ B) = n$ and $\mathrm{supp} A_i(\theta) \subset [-1+\varepsilon, 0]$ for some $\varepsilon > 0$.

We study the problem of exact null controllability as an infinite vector moment problem assuming that spectral controllability holds. The approach we used is essentially based on the property of minimality of the operator's family of exponentials. This allows to construct steering controls and solve moment problem for each state of the model space.

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Nano thermo-hydrodynamics models for quantitative estimations of the cell membrane fluidity: a review

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Nataliya Kizilova, *Kharkov, Ukraine*

The cells are the smallest units of the live matter, and their interaction with other cells and environment are determined by the cellular membranes. The latter possess mechanical, thermal and electric properties, which values strongly depend on the state of the cells (healthy, influenced, stresses, diseased). That is why the mathematical models of the cellular membrane and their physical properties are essential for the medical diagnostics purposes [1]. The mechanical properties of the cells and their membranes are represented by their density, elasticity and fluidity. While the density can be easily measured; the elasticity can be estimated by the rheometry and micro/nano indentometry; but the measurements of fluidity needs more complex mathematical models and governing equations for the heat transfer [2]. The most relevant model of the heat transfer at the micro and nano scales is based on the Navier-Stokes equations for the incompressive fluid (water as the main components of the cells and their membranes) combined by the heat transfer equation in the Guyer-Krumhansl form [3]

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} + \rho_b c_b w_b (T - T)_b) = k \left(\nabla^2 T + a \frac{\partial}{\partial t} \nabla^2 T \right) + q_m + q_e, \quad (1)$$

where T is the temperature, τ is the relaxation time, a is the diffusivity, k is the thermal conductivity, q_m and q_e are the methabolic and externally stimulated sources of heat, the subscript b related to the blood flow in the tissues provided the cells in the perfused tissue or bioreactor are considered.

The system of the Navier-Stokes equations together with the heat equation in the form (1) has been solved by the finite difference method with iterations over time.

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On synthesis problem for inherently nonlinear systems

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We study the controllability problem for a class of nonlinear systems of the form

$$\begin{cases} \dot{x}_1 = u, \quad |u(x)| \le d, \\ \dot{x}_i = x_{i-1}^{2k_{i-1}+1} + f_{i-1}(t,x,u), \quad i = 2,\dots,n, \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control, $d > 0$ is a given number, $k_i = \frac{p_i}{q_i}$ ($p_i > 0$ is an integer, $q_i > 0$ is an odd integer), $f_i(t,x,u)$ $(i = 1,\dots,n-1)$ are continuous real-valued functions with $f_i(t,0,0) = 0$ for all $t \ge 0$.

We construct a class of bounded controls $u = u(x)$ such that for any initial point $x_0 \in U(0)$ the solution $x(t,x_0)$ of the corresponding closed-loop system is well-defined on the interval $[0, T(x_0)]$ and ends at 0 in a finite-time $T(x_0) < +\infty$, i.e. $\lim_{t\to T(x_0)} x(t,x_0) = 0$.

The class of smooth stabilizing controls for system (1) was proposed in [1]. The synthesis problem for the case when $f_i(t,x,u) = 0$ $(i = 1,\dots,n-1)$ and $k_i = 0$ $(i = 1,\dots,n-2)$, $k_{n-1} > 0$ was solved in [2]. The approach which was proposed in [2] for constructing finite-time stabilizers is based on the controllability function method [3]. Under some additional growth conditions imposed on functions $f_i(t,x,u)$ we develop this approach to construct a class of bounded finite-time stabilizing controls $u = u(x)$ for system (1). To this end, we construct a class of controllability functions $\Theta(x)$ such that the inequality $\dot{\Theta}(x) \le -\beta\Theta^{1-\frac{1}{\alpha}}(x)$ holds for some $\alpha \ge 1$, $\beta > 0$. The former inequality guarantees that any trajectory of the closed-loop system starting in $U(0)$ hits the origin in some finite time $T(x_0)$.

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Constructive methods of investigation of the differential-algebraic Cauchy problem

Sergey Chuiko, *Slavyansk, Ukraine*

We investigate the problem of the determination of constructive conditions for the existence of solution $z(t) \in \mathbb{C}^1[a,b]$ of the linear differential-algebraic equation [1,2,3]

$$A(t)z'(t) = B(t)z(t) + f(t). \tag{1}$$

The matrices

$$A(t),\ B(t) \in \mathbb{C}_{m\times n}[a,b] := \mathbb{C}[a,b] \otimes \mathbb{R}^{m\times n}, \quad m \neq n$$

and the vector function $f(t) \in \mathbb{C}[a,b]$ are assumed to be continuous on the segment $[a,b]$.

Found solvability conditions and construction of the generalized Green operator of the Cauchy problem for a linear differential-algebraic system (1). Found sufficient conditions for reducibility generalized matrix differential-algebraic equation (1) to a sequence of systems combining differential and algebraic equations. An original classification is proposed, as well as a unified scheme for constructing solutions of differential-algebraic equations (1).

The method for construction of solvability conditions and construction of the generalized Green operator for linear differential-algebraic equation (1) can be generalized to boundary value problem for the matrix differential-algebraic equations in various critical and noncritical cases [4,5,6].

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[2] Boichuk A. A., Samoilenko A. M. Generalized Inverse Operators and Fredholm Boundary-value Problems 2-nd edition, Walter de Gruyter GmbH & Co KG, 2016: 1-298.

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Constructive methods of investigation of the differential-algebraic Cauchy problem with degenerate pulse action

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Elena Chuiko, *Slavyansk, Ukraine*

We investigate the problem of the determination of constructive conditions for the existence of solution [1]

$$z(t) \in \mathbb{C}^1\{[a;b] \setminus \{\tau_i\}_I\}$$

of the linear differential-algebraic equation [2,3]

$$A(t)z'(t) = B(t)z(t) + f(t), \quad t \neq \tau_i, \ i = 1, \ 2, \ \dots \ , \ p \tag{1}$$

with the impulse action [1,4]

$$\Delta z(\tau_i) = \mathcal{S}_i \ z(\tau_i - 0) + a_i, \quad \mathcal{S}_i \in \mathbb{R}^{n \times n}, \quad \tau_i \in [a,b], \quad a_i \in \mathbb{R}^n. \tag{2}$$

The matrices

$$A(t), \ B(t) \in \mathbb{C}_{k \times n}[a,b] := \mathbb{C}[a,b] \otimes \mathbb{R}^{m \times n}, \quad m \neq n$$

and the vector function $f(t) \in \mathbb{C}[a,b]$ are assumed to be continuous on the segment $[a,b]$. Provided

$$\det(I_n + \mathcal{S}_i) = 0, \ i = 1, \ 2, \ \dots \ , \ p$$

for the principal solution matrix $X(t)$ of the linear differential-algebraic equation (1) holds degenerate case [5,6].

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About an approximate solution of matrix differential-algebraic boundary-value problems with a least-squares method

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Olga Nesmelova, *Slavyansk, Ukraine*
Marina Dzuba, *Slavyansk, Ukraine*

We investigate the problem of the determination of conditions for the existence of solution [1]

$$Z(t) \in \mathbb{C}^1_{\alpha\times\beta}[a;b] := \mathbb{C}^1[a;b] \otimes \mathbb{R}^{\alpha\times\beta}$$

of the matrix differential-algebraic equation [2,3,4]

$$\mathcal{A}Z'(t) = \mathcal{B}Z(t) + F(t), \tag{1}$$

that satisfy the boundary condition

$$\mathcal{L}Z(\cdot) = \mathfrak{A}, \quad \mathfrak{A} \in \mathbb{R}^{\mu\times\nu} \tag{2}$$

and the construction of this solution. Here,

$$\mathcal{A}Z'(t): \ \mathbb{C}^1_{\alpha\times\beta}[a,b] \to \mathbb{C}_{\gamma\times\delta}[a,b], \ \ \mathcal{B}Z(t): \ \mathbb{C}^1_{\alpha\times\beta}[a,b] \to \mathbb{C}^1_{\gamma\times\delta}[a,b]$$

is a matrix operator, which ensures, by definition, the equality [5,6]

$$\mathcal{A}(\zeta'(t)\Xi_1 + \xi'(t)\Xi_2)(t) = \zeta'(t)\mathcal{A}(\Xi_1)(t) + \xi'(t)\mathcal{A}(\Xi_2)(t),$$

$$\mathcal{B}(\zeta(t)\Xi_1 + \xi(t)\Xi_2)(t) = \zeta(t)\mathcal{B}(\Xi_1)(t) + \xi(t)\mathcal{B}(\Xi_2)(t)$$

for any functions $\zeta(t), \xi(t) \in \mathbb{C}^1[a,b]$ and any constant matrices Ξ_1, Ξ_2.

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[2] Campbell S.L. Singular Systems of differential equations, Pitman Advanced Publishing Program, San Francisco-London-Melbourne, 1980: 1-178.

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Seminonlinear matrix boundary-value problem

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We establish necessary and sufficient conditions for the existence of solutions

$$Z(t,\varepsilon):\ Z(\cdot,\varepsilon)\in\mathbb{C}^1[a;b],\quad Z(t,\cdot)\in\mathbb{C}[0;\varepsilon_0],\quad Z(t,\varepsilon)\in\mathbb{R}^{\alpha\times\beta}$$

of a nonlinear matrix differential equation [1,2]

$$Z'(t,\varepsilon)=AZ(t,\varepsilon)+Z(t,\varepsilon)B+F(t,\varepsilon)+\varepsilon\ \Phi(Z(t,\varepsilon),\mu(\varepsilon),t,\varepsilon)\qquad(1)$$

with a boundary condition

$$\mathcal{L}Z(\cdot,\varepsilon)=\mathcal{A}+\varepsilon J(Z(\cdot,\varepsilon),\mu(\varepsilon),\varepsilon),\quad \mathcal{A}\in\mathbb{R}^{\delta\times\gamma},\ \ \alpha\neq\beta\neq\delta\neq\gamma.\qquad(2)$$

We seek the solution of the matrix boundary-value problem (1), (2) in a small neighborhood of the generating problem

$$Z_0'(t,\varepsilon)=AZ_0(t,\varepsilon)+Z_0(t,\varepsilon)B+F(t,\varepsilon),\quad \mathcal{L}Z_0(\cdot,\varepsilon)=\mathcal{A}.\qquad(3)$$

Here, $A\in\mathbb{R}^{\alpha\times\alpha}$ and $B\in\mathbb{R}^{\beta\times\beta}$ are constant matrices. Assume that the nonlinear matrix operator $\Phi(Z(t,\varepsilon),\mu(\varepsilon),t,\varepsilon):\ \mathbb{R}^{\alpha\times\beta}\to\mathbb{R}^{\alpha\times\beta}$ is Frechet differentiable with respect to the first argument in a small neighborhood of the solution of the generating problem and continuously differentiable with respect to μ in a small neighborhood of the solution of the generating problem (3) and the initial value $\mu_0(\varepsilon)$ of the eigenfunction $\mu(\varepsilon)$. The nonlinearity $\Phi(Z(t,\varepsilon),\mu(\varepsilon),t,\varepsilon)$ and inhomogeneity of the generating problem $F(t,\varepsilon)$ are regarded as continuous in t on a segment $[a,b]$ and in the small parameter ε on a segment $[0,\varepsilon_0]$. In addition, $\mathcal{L}Z(\cdot,\varepsilon)$ is a linear bounded matrix functional: $\mathcal{L}Z(\cdot,\varepsilon):\mathbb{C}^1[a;b]\to\mathbb{R}^{\delta\times\gamma}$. The nonlinear matrix functional $J(Z(\cdot,\varepsilon),\mu(\varepsilon),\varepsilon):C[a,b]\to\mathbb{R}^m$ is continuously differentiable with respect to Z in a small neighborhood of the solution of the generating problem (3), continuously differentiable with respect to μ in a small neighborhood of the solution of the generating problem (3) and the initial value $\mu_0(\varepsilon)$ of the eigenfunction $\mu(\varepsilon)$; and continuous in the small parameter ε on the segment $[0,\varepsilon_0]$.

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On the Stability of Invariant Sets of Functional Differential Equations with Delay

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Systems of functional differential equations with delay

$$dz(t)/dt = Z(t, z_t)$$

and

$$dz(t)/dt = Z(t, z_t) + R(t, z_t)$$

are considered where $z = (x, y), x \in \mathbb{R}^n, y \in \mathbb{R}^m$, and Z and R are the vector-valued functionals. It is supposed that these systems have a positive invariant set $x = 0$. The conditions are given when the uniform asymptotic stability of of the invariant set of the first system implies the uniform asymptotic stability of the invariant set of the second system. The asymptotic stability of this invariant set of the first system is studied separatly when the right-hand side of the system is an almost periodic in t.

Stability analysis of particular random switched linear dynamical system

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Interest in switched systems is based on their real-life applications. Engineering, natural and social systems cannot be simply described by a single model and in that cases systems exhibit switching between several models depending on various environments and applications. For example, switching has been extensively exploited in many engineering systems such as electronics, power systems, and traffic control.

The main reason of considering switch is that unpredictable, sudden change in the system dynamics or structures, such as a failure of a component or subsystem, or the accidental activation of any of the subsystems can occur. It is also introduced for effective control of highly complex non-linear systems of the so-called hybrid control. In both cases, an essential feature is the interaction between the continuous system dynamics and the discrete switching dynamics.

The main result of the work, given by simulations, confirms from a practical point of view the theoretical results [1],[2]. The examples of unstable and stable switched linear systems have been analysed and simulated with a help of the Matlab software - Simulink.

It was concluded that the stability of each linear system does not provide the stability of the corresponding switched linear system. Moreover, it was obtained that continuous-time switched linear system does not imply the stability in discrete-time.

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The synchronization of the angular velocities of identical rigid bodies

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We consider a mechanical system consisting of two rigid bodies, one of which is the master, and the other is the slave. It is assumed that the slave body has control, depending on its own state and the state of the leading body. We propose control law, that solves the problem of the bodies angular velocities synchronization in the form of feedback on the states of these systems. Our main goal to construct a control obtained from such feedback by substitution instead of the state of the master system their estimates obtained as a result observation problem solution. The question of whether such "approximate" control solve the initial problem are considered in stabilization theory, see for example, [1], where the corresponding separation principle was formulated. A non-linear observer is constructed using the method of invariant relations, the synthesis scheme of auxiliary invariant relations for which was described in [2]. Using the second Lyapunov method, it is shown that the output control thus obtained solves original synchronization problem.

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The Stieltjes matrix moment problem and associated positive symmetric operators

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A sequence of $m \times m$ matrices $(s_j)_{j=0}^{\infty}$ is called $\mathbb{R}$ - positive ($\mathbb{R}_+$ - positive) if, for all $l > 0$, the Hankel matrices $H_1^{(l)} = (s_{j+k})_{j,k=0}^{l}$ are positive(the Hankel matrices $H_1^{(l)}$ and $H_2^{(l)} = (s_{j+k+1})_{j,k=0}^{l}$ are positive, respectively).

Let the sequence $(s_j)_{j=0}^{\infty}$ be $\mathbb{R}$ - positive. Then nonnegative matrix measures σ are called *solutions* to the Hamburger matrix moment problem if

$$s_j = \int_{-\infty}^{\infty} t^j \sigma(dt), \quad j \geq 0.$$

Let the sequence $(s_j)_{j=0}^{\infty}$ be $\mathbb{R}_+$ - positive. Then nonnegative matrix measures σ are called *solutions* to the Stieltjes matrix moment problem if

$$s_j = \int_{0}^{\infty} t^j \sigma(dt), \quad j \geq 0.$$

The ranks of the radii of the limit Weyl discs are the geometric measure of degeneracy of the solution set to the Hamburger moment problem. The deficiency numbers of the associated symmetric operator are the operator measure of degeneracy of the solution set. Note that the geometric and operator measures of degeneracy are equal.

The ranks of the limit matrix Weyl intervals (see [1]) are the geometric measure of degeneracy of of the solution set to the Stieltjes moment problem. However, the operator measure of degeneracy of of the solution set to the Stieltjes matrix moment problem has remained uninvestigated. We introduce the operator measure of degeneracy of the solution set to the Stieltjes matrix moment problem in terms of the deficiency subspaces of a pair of positive symmetric operators, which is a novel approach. A relation between the operator and geometric measures of degeneracy of the solution set to the Stieltjes matrix moment problem is established. As a corollary, some results for the Stieltjes matrix moment problem are proved (see [2]).

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Controllability problems for the heat equation on a half-axis

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Consider the heat equation

$$w_t(x,t) = w_{xx}(x,t), \quad x \in (0,+\infty),\ w(0,t) = u(t), \tag{1}$$

controlled by the boundary condition

$$w(0,t) = u(t), \quad t \in (0,T), \tag{2}$$

under the initial condition

$$w(x,0) = w^0(x), \quad x \in (0,+\infty), \tag{3}$$

where $T > 0$ is given, $u \in L^\infty(0,T)$ is the control, the state $w(\cdot,t)$, $t \in (0,T)$, and the initial state w^0 belong to the space $H^0(0,+\infty)$ of the Sobolev type.

A state $w^0 \in H^0(0,+\infty)$ is called *approximately controllable* at a given time T if for any $w^T \in H^0(0,+\infty)$ and for any $\varepsilon > 0$ there exists a control $u_\varepsilon \in L^\infty(0,T)$ such that for the solution w_ε to system (1)–(3) with $u = u_\varepsilon$ we have $\left\|w^T - w_\varepsilon(\cdot,T)\right\| < \varepsilon$.

In the talk, it is shown that each state $w^0 \in H^0(0,+\infty)$ is approximately controllable at a given time T. The controls solving the approximate controllability problems are constructed.

For a state $w^0 \in H^0(0,+\infty)$, by $\mathcal{R}_T^1(w^0)$ denote a set of states $w^T \in H^0(0,+\infty)$ for which there exists a control $u \in L^\infty(0,T)$, $0 \le u(t) \le 1$, $t \in (0,T)$, such that for the solution w to system (1)–(3) we have $w(\cdot,T) = w^T$.

For states $w^0, w^T \in H^0(0,+\infty)$, we obtain necessary and sufficient conditions for $w^T \in \mathcal{R}_T^1(w^0)$. Under these conditions, using the Markov power moment problem, it is constructed a sequence $\{u_n\}_{n=1}^\infty$ of bang-bang controls ($u(t) \in \{0,1\}$, $t \in (0,T)$) such that for the solution w to system (1)–(3) with $u = u_n$ we have $\|w_n(\cdot,T) - w^T\| \to 0$ as $n \to \infty$.

These results are illustrated by examples.

Linear operator-differential equation with generalized quasipolinomial on the right-hand side

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We consider the Cauchy problem

$$u'(z) = Au(z) + e^{\gamma z} f(z), \quad z \in \mathbb{C}. \tag{1}$$

$$u(0) = b \in D(A), \tag{2}$$

where A is a closed operator on a complex Banach space X with a domain $D(A)$ ($D(A)$ is not necessarily dense in X), $\gamma \in \mathbb{C}$ and $f : \mathbb{C} \to X$ is an entire vector-valued function of zero exponential type.

Theorem 1. *If γ is a regular point of the operator A, then Equation (1) has the following unique solution of the form $e^{\gamma z} v(z)$, where $v(z)$ is an entire vector-valued function of zero exponential type,*

$$u(z) = -e^{\gamma z} \sum_{n=0}^{\infty} (A - \gamma I)^{-(n+1)} f^{(n)}(z).$$

Thus, the Cauchy problem (1), (2) has a solution of the above form if and only if $b = -\sum_{n=0}^{\infty} (A - \gamma I)^{-(n+1)} f^{(n)}(0)$.

Now, we assume that γ is an isolated point of the spectrum of A. We introduce the spectral projection P_γ corresponding to γ and expand the operator $A = A_\gamma \dot{+} \widetilde{A}_\gamma$ with respect to the direct sum $X = X_\gamma \dot{+} \widetilde{X}_\gamma$, $X_\gamma = P_\gamma(X)$, $\widetilde{X}_\gamma = (I - P_\gamma)(X)$.

Theorem 2. *If the operator $A - \gamma I$ is not quasinilpotent, then the Cauchy problem (1), (2) has a solution of the form $e^{\gamma z} v(z)$, where $v(z)$ is an entire vector-valued function of zero exponential type if and only if*

$$(I - P_\gamma) b = -\sum_{n=0}^{\infty} (\widetilde{A}_\gamma - \gamma I)^{-(n+1)} (I - P_\gamma) f^{(n)}(0).$$

Moreover, such a solution is unique and admits the representation

$$u(z) = e^{zA_\gamma} P_\gamma b + \int_0^z e^{\gamma \zeta} e^{(z-\zeta)A_\gamma} P_\gamma f(\zeta) d\zeta -$$

$$-e^{\gamma z} \sum_{n=0}^{\infty} (\widetilde{A}_\gamma - \gamma I)^{-(n+1)} (I - P_\gamma) f^{(n)}(z).$$

Approximate solutions of the Boltzmann equation with infinitely many modes

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Oleksii Hukalov, *Kharkiv, Ukraine*

We consider the nonlinear kinetic Boltzmann equation in case of a model of hard spheres [1]. We construct an approximate solution in the form

$$f(t,x,V)=\sum_{i=1}^{\infty}\varphi_i(t,x)M_i(V), \tag{1}$$

where $\varphi_i(t,x)$ are smooth, nonnegative and bounded on R^4 functions. The exact solutions $M_i(V)$ are global Maxwellians:

$$M_i(V)=\rho_i\left(\frac{\beta_i}{\pi}\right)^{3/2}e^{-\beta_i\left(V-\overline{V}_i\right)^2}.$$

We use the uniform-integral error:

$$\Delta=\sup_{(t,x)\in\mathbb{R}^4}\int_{\mathbb{R}^3}dV\Big|D(f)-Q(f,f)\Big|.$$

Theorem 1. *Let the coefficient functions $\varphi_i(t,x)$ in the distribution (1) be such that the functional series:*

$$\sum_{i=1}^{\infty}\varphi_iM_i,\quad \sum_{i=1}^{\infty}|V|\varphi_iM_i,\quad \sum_{i=1}^{\infty}M_i\left|\frac{\partial\varphi_i}{\partial t}\right|,\quad \sum_{i=1}^{\infty}M_i|V|\left|\frac{\partial\varphi_i}{\partial x}\right|$$

converge uniformly in the whole space R^4.

Then there exists such a quantity Δ', that $\Delta\leqslant\Delta'$ and $\lim\limits_{\beta_i\to+\infty}\Delta'$ is equal to:

$$\sum_{i=1}^{\infty}\rho_i\sup_{(t,x)\in\mathbb{R}^4}\left|\frac{\partial\varphi_i}{\partial t}+\left(\overline{V}_i,\frac{\partial\varphi_i}{\partial x}\right)\right|+2\pi d^2\sum_{\substack{i,j=1\\ i\neq j}}^{\infty}\rho_i\rho_j\Big|\overline{V}_i-\overline{V}_j\Big|\sup_{(t,x)\in\mathbb{R}^4}(\varphi_i\varphi_j).$$

The quantity Δ will be arbitrary small in the case, if $\varphi_i(t,x)=C_i(x-\overline{V}_it)$ or $\varphi_i(t,x)=E_i\left(\left[x,\overline{V}_i\right]\right)$ and with a special selection of hydrodynamic parameters.

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Feedback linearizability in the class C^1

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A control system $\dot{x} = f(x, u)$ is called feedback linearizable if it is reduced to a linear form $\dot{z} = Az + Bv$ by some change of variables $z = F(x)$ and a control $v = g(x, u)$. First results in the field were obtained in 1973. Namely, V. I. Korobov [1] introduced a special class of nonlinear systems ("triangular systems") which were feedback linearizable. These studies were originated by satellite control problems. Within this approach, triangular systems of the class C^1 were treated. On the other hand, A. Krener [2] considered the linearizability for affine systems of the class C^∞ by use of the Lie algebraic technique. Later, the linearizability problem in the class C^∞ was completely studied by B. Jakubczyk and W. Respondek [3] and other authors.

In [4] affine systems $\dot{x} = a(x) + b(x)u$ were considered where $a(x), b(x)$ are of the class C^1. It turned out that in this case the feedback linearizability conditions for systems of the class C^∞ [3] are neither necessary nor sufficient. The new ideas were proposed inspired by the original technique of triangular systems. In particular, it was proposed to use some other vector fields instead of Lie brackets which may not exist in the class C^1. In the talk we give an overview of the results of [4] and their further development [5]–[7].

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Representation of the organs' and tissues' regeneration processes as a solution of some optimal control problems the criteria and methods of which are derived from the biological principles of evolutionary developmental biology

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According to the most general ideas of the theoretical biology the regulatory of the processes of maintaining the organs' and tissues' dynamic homeostasis occurs due to the self-organization with some perturbation.

In complex dynamic systems of interacting cells the phenomenon of self-organization is determined by the structure and properties of the network of intercellular interactions. It is assumed that the dynamic system of interacting cells is in an unstable equilibrium in the critical point domain (phase transition).

The structure and properties of the network of intercellular interactions is the organ's and tissue's morphology which is inherited and, consequently, is subordinated to the natural selection in the evolution process.

Hypothesis

Regulatory of the processes of maintaining / restoring organs' and tissues' dynamic homeostasis on the basis of self-organization occurs according to certain principles, the criteria of optimality that have developed during the organism evolution.

On the example of liver regeneration we will consider the criteria for optimality and the possible structure of the control system by the regeneration processes proceeding from the principles of evolutionary development biology.

It is natural to assume that the model for a virtual control system is a deep neural network. It follows that regulatory according to certain criteria of optimality based on self-organization in the biological system of interacting cells is analogous to the neurodynamic programming methods [1].

The representation of the regulation of biological processes as the solution of some optimal control problems is one possible way of solving problems in mathematical cellular biology which are connected with enormous complexity, criticality and not observability.

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Trajectory optimization for underwater gliders

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The problem of finding of the shortest trajectory of underwater glider, which joins two oriented points with given constraints on curvature is presented. The underwater glider is driven only via system of actuators, which consists of an element, which is controlling the buoyancy of glider (called buoyancy engine) and moving batteries, which are controlling location of glider's center of gravity to maneuver. The absence of any other thrusts, screws or engines leads to strong influence of hydrodynamic forces on its movement and, as a result, to the complicated nonlinear mathematical model.

First time, a similar class of problems was investigated by A. A. Markov (1889) in case of railway projection [1]. Later, L. E. Dubins' results on this problem [2], presented in 1957, were widely applied to a cars' motion. Nowadays, the revival of interest in such problems is associated with tremendous number of applications in robotics, for example in the theory of motion of autonomous underwater vehicles.

The report presents an overview of the main works which generalize the solutions of Markov-Dubins problem which were obtained until now. In modern works, since J.-D. Boissonnat's report in 1991 [3], the Pontryagin's Maximum Principle is using as a foundation to solve the problem.

In this paper we consider new problem statements that take into account the specific dynamics of a rigid body in dense incompressible fluid.

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On numerical modeling of the river flows with validation on the measurement data

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Mathematical modeling of the river flows in the natural complex geometries is a challenging problem for the environmental mechanics. Recently the numerical simulations of the river flows is mostly based on the diffusive wave approximation of the Saint-Venant equations which are derived by integration over the river depth of the Reynolds averaged Navier-Stokes equations [1]. It is accepted, the variations in the river morphology are small, and the empirical flow resistance distributions are accounted for. These equations are hyperbolic and capable of studying some extreme conditions like the dam breaks. The diffusive wave approximation of the Saint-Venant equations is parabolic and is obtained by neglecting the inertial terms, so they are restricted to subcritical flow conditions. The diffusive wave shallow water equation is

$$\frac{\partial h}{\partial t} + div(\vec{q}) = q_c, \tag{1}$$

where $h = H + z$ is the hydrological head, H is the river depth, z is the coordinate of the river surface, q_c are sources of water along the river bed, $\vec{q} = -kK\nabla h$ is the flux, $k = H^{5/3}$ is the conductivity, $K = m^{-1}\sqrt{S}$ is the resistivity of the river bed, $S = \left((\partial h/\partial x)^2 + (\partial h/\partial z)^2\right)^{1/2}$ is the friction slope, m is the Manning coefficient. The reliability of the approximation (1) for the runoff simulations has been shown in a series of research papers [1]. Here the model (1) has been used for the numerical computations of the velocity, pressure, and vortexes distributions in a segment of the Seversky Donets River near Kharkiv city. The results of the FEM computations on the model has been compared to the measurement data in dynamics during the 2015 – 2018 years. The flow-based bottom modifications are explained by the mechanical forces and the ratio between the areas of the moving waters and stagnant waters has been proposed for the prognosis of evolution of the river basin.

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Spectral properties of a non-selfadjoint differential operator with block-triangular operator coefficients

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In the study of the connection between spectral and oscillation properties of non-self-adjoint differential operators with block-triangular operator coefficients (see [1]) the question arises of the structure of the spectrum of such operators. In spite of the fact that the differential operator with block-triangular coefficients is non-self-adjoint, under certain conditions its spectrum can be real. At the same time, a non-self-adjoint operator, unlike a self-adjoint operator, can have points at which the resolvent has a pole, but which are not eigenvalues of the operator. They are called spectral singularities.

For an operator with a triangular matrix potential decreasing at infinity, which has a bounded first moment, the structure of the spectrum was established in the works of F.S. Rofe - Beketov and E.N. Bondarenko.

In this work we have obtained sufficient conditions under which the spectrum of a non-self-adjoint differential operator with block- triangular operator potential growing at infinity is real and discrete. The operator has no spectral singularities and its spectrum coincides with the union of the spectra of semibounded self-adjoint operators corresponding to self-adjoint diagonal elements. In this case, the growth rate of elements not standing on the main diagonal is subordinated to the growth rate of the diagonal elements. If these conditions are violated, the appearance of points of spectral singularities is possible. An example is given in [2].

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On bioheat equation and its modifications

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Mathematical modeling of the optical and infrared heating of biological tissues is based on the Pennes bioheat balance equation [1] and its modifications, the single-phase lag (SFL) [2] and dual-phase lag (DFL) [3] models. The Pennes bioheat equation is

$$\rho c\frac{\partial T}{\partial t} = div\,(k\nabla T) + q_{met} = q_h + \rho_b c_b w_b (T_b - T), \tag{1}$$

where T is the temperature, ρ, c, k are the density, specific heat and thermal conductivity of the tissue, the subscript b relates to the blood, w_b is the blood perfusion rate, q_{met} and q_h are metabolic and photostimulated heats.

The SFL model accounts for the time delay τ_q between the heat flux q and the temperature gradient ∇T that give the equation

$$\rho c\tau_q \frac{\partial^2 T}{\partial t^2} + (\rho c + \rho_b c_b w_b \tau_q)\frac{\partial T}{\partial t} = div\,(k\nabla T) + q_{met} + q_h + \rho_b c_b w_b T_b. \tag{2}$$

The DTL model accounts for two time lags τ_q and τ_T and has the form similar to (2). As it was shown, the SFL and DFL models are thermodynamically inconsistent, while the Guyer-Krumhansl equation as an example of the non-Fourier heat conduction law is thermodynamically correct.

In this paper the 1D solutions of the models (1)-(2) and the Guyer-Krumhansl equation for the surface heating of human skin are considered. The computational results are compared to the measured curves $T(t)$. It is shown, the Guyer-Krumhansl equation gives the best correspondance between the computational and measured curves for both heating and relaxation precesses.

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Mathematical modeling of the bioactive arterial wall

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Due to their bioactivity, the vessel walls can respond to the elevation of the blood pressure and wall shear stress [1]. The mechanical model of the bioactive arterial wall is based on the rheological equation of the wall [1]

$$\Lambda_R \frac{\partial R}{\partial t} + R = \Lambda_P \frac{\partial P}{\partial t} + \left(F_1(P) - F_2(C)\right)\Phi(b), \tag{1}$$

where C and b are concentrations of Ca^{++} and NO, R and P are the radius of the vessel and the blood pressure in it, Λ_R, Λ_P, $F_1(P)$, $F_1(P)$ and $\Phi(b)$ are known empirical functions.

The influence of the Ca^{++} on the smooth muscle cells is governed by the kinetic equation [2]

$$\alpha \frac{\partial C}{\partial t} = -C + \psi(\sigma) + \beta \frac{\partial P}{\partial t}, \tag{2}$$

where $\sigma = PR/h$ is the circumferential stress, h is the wall thickness, α and β are constants.

Distribution of the NO is givерned by the diffusion equation

$$\frac{\partial b}{\partial t} = D_b \nabla^2 b - k b^{\xi}, \tag{3}$$

where D_b is the diffusion coefficient, k and ξ are constants.

The momentum equation for the wall has been taken in the form [2]

$$2\pi R\frac{\partial R}{\partial t} = \frac{\pi}{8\mu}\frac{\partial}{\partial t}\left(R^4\frac{\partial P}{\partial x}\right), \tag{4}$$

where μ is the blood viscosity.

The system of partial differential equations (1)-(4) has been studied numerically by the finire difference method and iterations over time at a wide set of material parameters. Different regimes of the flow control by the bioactive wall are discussed.

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Feedback synthesis for motion of a material point with allowance for friction

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Let us consider the feedback synthesis for motion of material point with allowance for friction:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ p(t,x_1,x_2)x_2 + u \end{pmatrix}. \tag{1}$$

Here $t \geq 0$, $(x_1,x_2) \in \mathbb{R}^2$ is a state; u is a scalar control, $|u| \leq 1$; $p(t,x_1,x_2)$ is *unknown* nonlinear viscous friction, $p_1 \leq p(t,x_1,x_2) \leq p_2$. The approach presented in the talk is based on the controllability function method proposed by V.I. Korobov in 1979. In [1] a control $u(x)$ solving the feedback synthesis problem for system (1) without friction was given. It satisfies two conditions: 1) $|u(x)| \leq 1$; 2) the trajectory $x(t)$ starting from an initial point $x(0) = x_0 \in \mathbb{R}^2$ of the closed system $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ u(x) \end{pmatrix}$ ends at the origin at a finite time $T > 0$, and, in addition, the time T is equal to $\Theta(x_0)$ for any $x_0 \in \mathbb{R}^2$.

The *main goal* of the research is to find friction limits such that a control steering the system without friction to the origin also steers the system with friction to the same target.

Theorem 1. *Let* $a_1 < -4.5$, $0 < \gamma_1 < 1$, $\gamma_2 > 1$, $c > 0$. *The controllability function* $\Theta = \Theta(x_1,x_2)$ *is defined for* $x \neq 0$ *as a unique positive solution of the* *equation*

$$\frac{(4+a_1)\Theta^4}{a_1(3+a_1)} - a_1x_1^2 + 4\Theta x_1x_2 + \Theta^2 x_2^2 = 0. \qquad (2)$$

Let $Q = \{(x_1,x_2) \mid \Theta(x_1,x_2) \le c\}$, $\quad u(x) = \dfrac{a_1x_1}{\Theta^2(x_1,x_2)} - \dfrac{3x_1}{\Theta(x_1,x_2)}$,

$$p_1^0 = \max\{(1-\gamma_1)\tilde{p}_1^0; (1-\gamma_2)\tilde{p}_2^0\}, \qquad p_2^0 = \min\{(1-\gamma_1)\tilde{p}_2^0; (1-\gamma_2)\tilde{p}_1^0\},$$

$$\tilde{p}_1^0 = \frac{3+a_1-\sqrt{a_1(a_1+4)}}{c}, \qquad \tilde{p}_2^0 = \frac{3+a_1+\sqrt{a_1(a_1+4)}}{c}.$$

Then, for all $p_1 \le p(t,x_1,x_2) \le p_2$ *such that* $[p_1;p_2] \subset (p_1^0;p_2^0)$, *the trajectory of the closed system starting from an initial point* $x(0) = x_0 \in Q$ *ends at the point* $x(T) = 0$ *at a finite time* $T = T(x_0,p_1,p_2)$ *under the estimate* $\Theta(x_0)/\gamma_2 \le T(x_0,p_1,p_2) \le \Theta(x_0)/\gamma_1$.

Furthermore we analyze the envelope for one-parametric family (2) at $\Theta = 1$ for the system (1) with $p(t,x_1,x_2) = 0$. It is close to the curve that describes all points from which we may steer to the origin due to the Pontryagin maximum principle for the time $t = 1$.

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On one class of non-dissipative operators

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The study of the basicity of systems of functions, as a rule, is based on the study of some properties of linear operators. The study of the so-called class of quasi-exponentials provokes special interest, it is started by B. S. Pavlov and then developed and continued by S. V. Hruščev, N. K. Nikolsky, B. S. Pavlov. An approach suggested by G. M. Gubreyev is an important method of studying problems of basicity in this realm of analysis. He succeeded in harmonic combination of deep problems of spectral analysis of non-selfadjoint operators and delicate analytical results of the theory of functions. This work is a development of ideas of the paper by G. M. Gubreyev and V. N. Levchuk in which the study of Dunkl kernels is based on the analysis of a non-selfadjoint operator with two-dimensional imaginary component. (The function $d_\alpha(\lambda) = 2^\alpha\Gamma(\alpha+1)\lambda^{-\alpha}\left(J_\alpha(\lambda) + iJ_{\alpha+1}(\lambda)\right)$ is said to be a Dunkl kernel, where $J_\alpha(\lambda)$ is a Bessel function.) In contrast to, here the power dependence of the weight function is not supposed. This work is dedicated to the study of one class of

Volterra non-dissipative operators and to the construction of model representations for them. It turns out that many statements from are general and can be obtained for "arbitrary" weight functions $\varphi(x)$. General properties of the operator B are studied ant its characteristic function is calculated. Calculation of this characteristic function is based on the solution of the equation of the second order which depending on the choice of the weight $\varphi(x)$ turns into either a Bessel equation, a Mathieu equation, or a Lamé equation. Similarity of the studied non-dissipative operator to the operator of integration in the space of quadratically summed functions on $[-a, a]$ is proved. A functional model of a non-dissipative operator in the L. de Branges space is listed, and it is shown that in the special case, when $\varphi(x) = x^\nu$, the Dunkl kernels "coincide" with $E(\lambda)$.

Controllability of second-order partial differential equations in time

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Consider the following Cauchy problem

$$\begin{gathered}\frac{\partial^2 w(x,t)}{\partial t^2} + P\left(\frac{\partial}{\partial x}\right)\frac{\partial w(x,t)}{\partial t} + Q\left(\frac{\partial}{\partial x}\right) w(x,t) = u(x)v(t), \\ w(x,0) = \varphi(x), \ \ w'_t(x,0) = \psi(x),\end{gathered} \tag{1}$$

where $P\left(\frac{\partial}{\partial x}\right)$ and $Q\left(\frac{\partial}{\partial x}\right)$ — differential operators with constant coefficients, $v(t)$ — piecewise continuous function on a segment $[0;T]$ and functions $u(x)$, $\varphi(x)$, $\psi(x)$ belong to the Schwartz space S. We seek for a control $u(x)v(t)$ such that for all $t \in [0;T]$ the solution $w(x,t)$ belongs to S and condition $w(x,T) = 0$ is fulfilled.

Suppose $\lambda_1(s)$ and $\lambda_2(s)$ are roots of the characteristic equation $\lambda^2 + P(is)\lambda + Q(is) = 0$.

Then the Cauchy function for the Fourier transformed equation $\widetilde{w}''_{tt}(s,t) + P(is)\widetilde{w}'_t(s,t) + Q(is)\widetilde{w}(s,t) = \widetilde{u}(s)v(t)$ is as follows:

$$K(s,t,\tau) = \frac{\left(e^{\lambda_1(s)(t-\tau)} - e^{\lambda_2(s)(t-\tau)}\right)}{\lambda_1(s) - \lambda_2(s)}.$$

The controllability condition of equation (1) will look like this $R(s,T) = \int_0^T K(s,T,\tau)v(\tau)d\tau \neq 0, \ \forall s \in \mathbb{R}^n$. The following results are valid.

Theorem 1. *If the roots of the characteristic equation $\lambda_j(s)$ are real and at least one of them is bounded from above, then a control with $v(t) = 1$ and $u(x) \in S$ such that equation (1) is controllable in the space S exists.*

Theorem 2. *If the roots of the characteristic equation $\lambda_j(s)$ are imaginary, then a control of the form $e^{\gamma(T-t)}u(x)$, where $u(x) \in S$, such that equation (1) is controllable in the space S with some $\gamma > 0$ exists.*

Theorem 3. *If conditions $Q(is) = 0$ and $ReP(is) \leq c$ $\forall s \in \mathbb{R}^n$ are satisfied, then equation (1) is controllable in the space S with $v(t) = 1$.*

`Example`. Equation $\frac{\partial^2 w(x,t)}{\partial t^2} - a^2 \triangle w(x,t) - kw(x,t) = u(x)v(t)$ is controllable in the space S with control of the form $e^{\gamma(T-t)}u(x)$ with some $\gamma > 0$.

Resolvent for certain classes of generators of C_0-groups

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The spectral theory of nonselfadjoint operators is much more complicated than the theory for selfadjoint ones and it has many open problems. This is caused mainly by a fact that the spectrum does not contain much information about the behavior of nonselfadjoint operator. Thus important problems are obtaining the explicit form of the resolvent and controlling the resolvent of nonselfadjoint operator.

In this talk we will discuss the explicit form and asymptotic properties of the resolvent for the classes of generators of C_0-groups with purely imaginary eigenvalues, clustering at $i\infty$, and complete minimal family of eigenvectors which, however, do not form a Schauder basis. These classes were recently presented by the author and Grigory Sklyar in [1]. Multiple applying of the discrete Hardy inequality serves as the keystone for the proofs of the corresponding results.

This is a joint work with Grigory Sklyar.

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Remarks on lower semicontinuous solutions of Hamilton-Jacobi-Bellman equations

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This talk is devoted to lower semicontinuous solutions of Hamilton-Jacobi equations with convex Hamiltonians in the gradient variable. Such Hamiltonians do arise in the optimal control theory. We present a necessary and sufficient condition for the reduction of the Hamiltonian satisfying optimality conditions to the case when the Hamiltonian is positively homogeneous and also satisfies optimality conditions. On one hand it allows us to reduce uniqueness of solutions problem to Barron-Jensen [1] and Frankowska [2] theorems. On the other hand it shows us the limits of applicability of this reduction. For Hamiltonians which are not subject to the above reduction we present the new existence and uniqueness theorems.

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On the integration of nonlinear differential equation

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The Lax system of this form is studied in this paper

$$\begin{cases} [a(x), \gamma(x)] = 0, \quad x \in [0, l], \\ \gamma'(x) = i[a(x), \sigma_2], \quad x \in [0, l], \\ \gamma(0) = \gamma^+, \end{cases} \tag{1}$$

where $a(x)$ – spectral matrix measure, $\gamma(x), \sigma_2, \gamma^+$ – self-conjugate $n \times n$ matrices, and

$$a(x) \geqslant 0, \qquad tra(x) \equiv 1, \qquad x \in [0, l].$$

The solution of this system $\gamma(x)$ is used in construction of triangular models of commutative systems of operators [1].

Proposition 1. *Let* $\sigma_2 = diag(b_1, ..., b_n), \quad \gamma^+ = \alpha_1\sigma_2 + \alpha_0 I + iC$, *where* $\alpha_1, \alpha_0 \in \mathbb{R}$, *matrix* $C = (c_{jk})_{j,k=1}^n = -C^*$ *and* $c_{jj} = 0$, $j \in \{1, \dots, n\}$.

Let further $\kappa_0,\ \kappa_1,\ \kappa_2 \in L^1[0, l]$ *– are real-valued functions. Then pair* $\{a(\cdot), \gamma(\cdot)\}$, *where* $a(x) = \kappa_2(x)\gamma(x)^2 + \kappa_1(x)\gamma(x) + \kappa_0(x)$, $x \in [0, l]$, *and* $\gamma(\cdot) = (\gamma_{jk}(\cdot))_{j,k=1}^n$, *is the solution of the (1) if and only if* $x \in [0, l]$ *the following equations are completed*

$$\begin{aligned} \gamma_{jj}(x) &= \gamma_{jj}^+, \quad j \in \{1, \dots, n\}, \\ \gamma_{jk}(x) &= ie^{i(b_j - b_k)\left(K_1(x) + (\gamma_{jj}^+ + \gamma_{kk}^+)K_2(x)\right)} y_{jk}(x), \quad j \neq k, \end{aligned}$$

where

$$K_j(x) := \int_0^x \kappa_j(t)dt, \quad j \in \{1, 2\},$$

and the functions $y_{jk}(\cdot)$, $j \neq k$, satisfy the system

$$\begin{cases} y'_{jk}(x) & = (b_k - b_j)\kappa_2(x) \sum\limits_{s=1, s\neq j,k}^{n} y_{js}(x) y_{sk}(x), \quad x \in [0,l], \quad j \neq k, \\ y_{kj}(x) & = \overline{-y_{jk}(x)}, \quad x \in [0,l], \quad j \neq k, \\ y_{jk}(0) & = c_{jk}, \quad j \neq k. \end{cases} \tag{2}$$

At that, if $c_{jk} \in \mathbb{R}$, $j \neq k$, then any solution of the system (2) is real-valued.

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On asymptotic growth of solutions of C_0 semigroups

Piotr Polak, *Szczecin, Poland*
Grigorij Sklyar, *Szczecin, Poland*

We consider linear differential equation

$$\dot{x}(t) = Ax(t), \quad x(t) \in D(A) \subset X,$$

where $A : D(A) \to X$ is a closed (usually unbounded) operator generating C_0 semigroup $\{T(t)\}_{t\geq 0}$ on Banach space X. The talk is devoted to some aspects of stability of the semigroup $T(t)$ and corresponding solutions $T(t)x$. We discuss some spectral conditions of asymptotic stability and present the generalizations of stability concept: polynomial stability and the existence of the fastest growing solution - so called maximal asymtotics. In particular we present our results in the field of asymptotic behaviour of strongly continuous semigroups: theorem on the sufficient condition for polynomial stability and theorem on non existence of maximal asymptitics for some types of semigroups acting on a Banach space.

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Category-theoretic Methods for Studying Causality in Distributed Systems

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Grygoriy Zholtkevych, *Kharkiv, Ukraine*

The report describes the category-theoretic approach to the study of causality in discrete distributed systems. The trend of widespread use of distributed computing, observed in recent years, is a technological answer to the practical achievement of the upper bound of processor performance on the one side and the development of communication tools on the other. In addition, there is a tendency to integrate cybernetic and physical systems, which has been accelerated in the context of developing Internet-of-Things.

The analysis of the problem allows us to state that the problems associated with controlling parallel, distributed and concurrent computations turned out to be on the leading edge of Computer Science and Information Technology. In the focus of studying this problem area, the problem of modelling causality in distributed systems holds a central position, in particular, modelling based on the concept of logical time. There are two approaches to model logical time, namely, the event-based approach and the state-based approach. Unfortunately, the set-theoretic language does not give a natural description of the relationship between these approaches. Our research focuses on the use of the language of category theory, which is adequate for constructing models, is based on events and is based on states.

There are a lot of studies deal with controlling discrete event dynamic systems, which use the category-theoretic languages. These studies use the category-theoretic notion "adjunction" to describe the interrelation between event-based and state-based models of the systems.

Our main results are as follows

1. the category of clock structures has been defined; this category is used to define event-based models of logical time;
2. the subcategory of linear clock structures in the category of clock structures has been defined; this category is used to define physical models of logical time;
3. the category of schedules has been defined; this category is a bridge between event-based and state-based modelling approaches;
4. equivalence between the categories of linear clock structures and schedules has been proven.

Stopping of oscillations of controlled elliptic pendulum

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Vladislav Chuiko, *Kharkiv, Ukraine*

This work deals with the feedback synthesis problem for controlled elliptic pendulum. The main idea of the article is to find the control that will steer the initial point to the origin. The equations describing the motion of a controlled elliptic pendulum were constructed. They are provided below.

$$\begin{cases} \ddot{y} = -\dfrac{glm_2\varphi + lv_1 - v_2}{lm_1}, \\ \ddot{\varphi} = \dfrac{-\varphi glm_2(m_1+m_2) - lm_2v_1 + (m_1+m_2)v_2}{l^2m_1m_2}, \end{cases} \tag{1}$$

where $y(t)$ is the horizontal axis deflection and $\varphi(t)$ is the the deviation from the bottom stability stance. The method of controlability function invented by V.I. Korobov was used. Firstly, using the next change of variables:

$$z_1 = \varphi, z_2 = \dot{\varphi}, z_3 = y, z_4 = \dot{y} \tag{2}$$

we've transformed the 2-dimensional system into the linear 4-dimensional system. After that, we have transformed it to the canonical system using the linear change of variables $x = Lz$. Using abovementioned method, bounded control that stops the oscillation of this mechanical system was constructed. The controllabiblity function $\Theta(x)$ is defined as a unique positive solution of the equation

$$2a_0\Theta^4 = 36x_1^2 + 24\Theta x_1x_2 + 6\Theta^2x_2^2 + 36x_3^2 + 24\Theta x_3x_4 + 6\Theta^2x_4^2, \tag{3}$$

for some $a_0 > 0$. The next equality should be used to find the necessary control.

$$v(x) = \begin{pmatrix} v_1(x) \\ v_2(x) \end{pmatrix} = \begin{pmatrix} -\dfrac{6x_1}{\Theta^2(x)} - \dfrac{3x_2}{\Theta(x)} \\ gx_1 + x_3\dfrac{g(m_1+m_2)}{lm_1} - \dfrac{6x_3}{\Theta^2(x)} - \dfrac{3x_4}{\Theta(x)} \end{pmatrix} \tag{4}$$

This control steers an arbitrary initial point of a certain neighborhood of the origin of the coordinates to the origin in a finite time. Graphics of the trajectory and control on the trajectory, which begin from the chosen initial point, are presented.

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Solutions to nonlinear systems of reaction-diffusion equations/ ODEs with delay

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We are interested in a class of systems of non-linear partial differential equations/ODEs with different types of bounded time delays. To describe systems, we remind the notation which is usual in the theory of delay equations. Considering the maximal delay $h > 0$, for a function $v(t), t \in [a-h, b] \subset \mathbb{R}, b > a$, we denote the history segment $v_t = v_t(\theta) \equiv v(t+\theta), \theta \in [-h, 0], t \in [a, b]$.

The general form of a delay system under consideration is

$$\frac{d}{dt}u(t) + Au(t) = F(u_t), \tag{1}$$

In (1), A is an unbounded linear operator in a Banach space X, $F : C \equiv C([-h, 0]; X) \to X$ is a nonlinear (delay) map. The form of F depends on particular applied problems. Initial conditions, in general, are

$$u|_{[-h,0]} = \varphi \in C \equiv C([-h, 0]; X). \tag{2}$$

For particular cases, the set of initial functions φ could be a carefully choosen subset (not necessarily linear) of the space C.

We are interested in reaction-diffusion systems in bounded domains with different types of delay in reaction terms. Particular interest is in the case of presence of discrete state-dependent delays. This type of delay is the most relevant to real-world applications and most difficult from mathematical point of view. For a survey on the ODE theory see [1]. The well-posedness in the sense of Hadamard and long time asymptotic behaviour of different types of solutions to (1)-(2) are studied (see e.g. [2, 3, 4]).

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Application of the method of lines to discretize problems of controllability for the partial differential equations, representing processes in power installations

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Suitable controlling programs, providing required state parameters during an operation in different modes, must be constructed for power installations, and it is an interesting not fully explored area for using the controllability theory.

Mathematical models of processes in power installations, including the heat conduction and others, can be represented using partial differential equations. In a point $\mathbf{x}$ we have a state vector $\mathbf{y}(\mathbf{x}, t)$, changing during a time $t \geq 0$ from an initial state $\mathbf{y}_0$ in corresponding with the controlling program, represented by a vector $\mathbf{u}(t)$, properties of the process in a domain Ω and an environment influence at a boundary Γ, represented by operators $\mathbf{A}(\mathbf{y}, \mathbf{u})$ and $\mathbf{B}(\mathbf{y}, \mathbf{u})$:

$$\partial \mathbf{y}/\partial t = \mathbf{A}(\mathbf{y}, \partial \mathbf{y}/\partial \mathbf{x}, \mathbf{u}),\ \mathbf{y}(x, 0) = \mathbf{y}_0\, \mathbf{x} \in \Omega, \quad \mathbf{B}(\mathbf{y}, \mathbf{u}) = 0\, \mathbf{x} \in \Gamma. \tag{1}$$

Mathematical model (1) is necessary to build the controlling program $\mathbf{u}(t)$, allowing to change the initial state of the process to a given state $\mathbf{y}_T$ during a minimal time T under required limiting conditions, represented in an operator $\mathbf{C}(\mathbf{y}, \mathbf{u})$:

$$\mathbf{u}(t): \quad \mathbf{y}(\mathbf{x}, T) = \mathbf{y}_T,\ \mathbf{C}(\mathbf{y}, \mathbf{u}) \geq 0,\ T \to \min. \tag{2}$$

To reduce the problem (1), (2), we use the spatial grid with nodes $\mathbf{x}_k \in \Omega$ and nodal values $\mathbf{y}_k(t) = \mathbf{y}(\mathbf{x}_k, t)$, $k = 1, 2, ..., n$. Following the method of lines [1], we use a finite differences technique in nodes $\mathbf{x}_k \in \Omega$ only for the differential operator $\partial \mathbf{y}/\partial \mathbf{x}$ and we reduce the problem (1), (2) to a view:

$$d\bar{\mathbf{y}}/dt = \bar{\mathbf{A}}(\bar{\mathbf{y}}, \mathbf{u}),\ \bar{\mathbf{y}}(0) = \bar{\mathbf{y}}_0,\ \mathbf{u}(t) : \bar{\mathbf{y}}(T) = \bar{\mathbf{y}}_T,\ \bar{\mathbf{C}}(\bar{\mathbf{y}}, \mathbf{u}) \geq 0,\ T \to \min, \tag{3}$$

where $\bar{\mathbf{y}}$ is a vector, including the nodal values $\mathbf{y}_1,\ \mathbf{y}_2,\ ...,\ \mathbf{y}_n$

Thus, the method of lines give us the opportunities to discretize the problem of controllability for partial differential equations, representing processes in power installations, and allows to reduce it to the controllability of the ordinary differential equations considered, for example, in [2].

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The integrable nonlocal nonlinear Schrödinger equation: Riemann-Hilbert approach and long-time asymptotics

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We study the initial value problem for the integrable nonlocal nonlinear Schrödinger (NNLS) equation

$$iq_t(x,t) + q_{xx}(x,t) + 2q^2(x,t)\bar{q}(-x,t) = 0$$

with decaying (as $x \to \pm\infty$) boundary conditions as well as with the step-like boundary conditions: $q(x,0) \to 0$ as $x \to -\infty$ and $q(x,0) \to A$ as $x \to -\infty$, where $A \neq 0$.

The main aim is to describe the long-time ($t \to +\infty$) behavior of the solution of these problems. To do this, we adapt the nonlinear steepest-decent method to the study of the Riemann-Hilbert problem associated with the NNLS equation. In the case of decaying initial data, our main result is that, in contrast to the local NLS equation, where the main asymptotic term (in the solitonless case) decays to 0 as $O(t^{-1/2})$ along any ray $x/t = const$, the power decay rate in the case of the NNLS depends, in general, on x/t, and can be expressed in terms of the spectral functions associated with the initial data.

In the case of the step-like boundary conditions, the asymptotics turns to be different in different sectors of the (x,t) plane. Particularly, in the right-most sector, the main asymptotic terms is a constant depending on the ratio x/t whereas the second term decays, as in the previous case, with the power decay rate depending on x/t.

Vanishing of solution of the model representative of NPE

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The theory of quasilinear parabolic equations has been developed since the 50-s of the 19th century. The properties of these equations differ greatly from those of linear equations. These differences were revealed in the scientific papers of the mathematicians: Barenblatt G.I., Oleinic O.A., Kalashnikov A.S., Zhou Yu Lin and others. Specific properties of NE (inertia, strongweaked localization of solutions' supports, extinction...) were studied by J.I. Diaz, L. Veron, A.E. Shishkov, B. Helffer, Y. Belaud, D. Andreucci and others. The most important aspect of such investigations is the description of structural conditions affecting the appearance and disappearance of various non-linear phenomena. Our investigation deals with nonlinear parabolic equation with degenerating absorption potential h(t), the presence of which play the important role in the study of the above mentioned properties.

So, we study Cauchy-Neumann problem for the next type of a quasilinear parabolic equation with the model representative:

$$u_t - \Delta u + h(t)|u|^{q-1}\, u = 0 \ \text{ in}\, \Omega \times (0, T) \tag{1}$$

$$\frac{\partial u}{\partial n}|_{\partial\Omega\times[0,T]} = 0 \tag{2}$$

$$u(x, 0) = u_0(x), \quad \mathbb{R}^N \backslash \{\mathrm{supp}\, u_0\} \neq \emptyset, \{\mathrm{supp}\, u_0\} \subset \{|x| < 1\} \tag{3}$$

Here $0 < q < 1$, the initial function $u_0(x) \in L_2(\Omega)$, $\Omega \subset \mathbb{R}^N (N \geqslant 1)$ be a bounded domain with C^1 - boundary. Assume that h(t) is a continuous, nonnegative, nondecreasing function, such that $h(0) = 0$. Let $h(t) = exp(-\frac{\omega(t)}{t})$, where $\omega(t)$ satisfies following technical conditions: (A) $\omega(t) > 0 \ \ \forall\, t > 0$, (B) $\omega(0) = 0$, (C) $\frac{t\,\omega'(t)}{\omega(t)} \leq 1 - \delta \ \ \forall\, t \in (0, t_0)$, $t_0 > 0$, $0 < \delta < 1$.

Theorem 1. *Let be an arbitrary function from $L_2(\Omega)$, $\omega(t)$ is continuous and nondecreasing function satisfy assumptions (A)(B)(C), then an arbitrary solution $u(x, t)$ of the problem (1)(2)(3) vanishes on Ω in some finite time $T < \infty$.*

To prove that, we use local energy method, which deals with norms of solutions $u(x, t)$ only and, therefore, may applied for higher order equations too.

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One optimal control problem for an unmanned aerial vehicle

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The papers [1, 2] deal with one problem of minimizing the time for a kinematic model of unmanned aerial vehicle moving at a constant altitude. From a kinematic point of view, an UAV flying at a constant altitude is determined by standard Dubins equations [3]. Under additional speed constraints, the flight model of a drone is described by the following system of differential equations:

$$\dot{x} = \cos\theta, \quad \dot{y} = \sin\theta, \quad \dot{\theta} = u, \tag{1}$$

with $(x, y, \theta) \in \mathbb{R}^2 \times \mathbb{S}^1$ (where $(x, y) \in \mathbb{R}^2$ is UAV coordinates in the plane of constant height, θ is the angle of deviation from the course) and the control $u \in [-1, 1]$. In [1, 2] this (and more general) time-optimal problem was considered with the following final conditions: the UAV steers to the circle of radius 1 centered at the origin and then moves along it clockwise. Due to such final conditions, choosing a new basis $(\widetilde{x}, \widetilde{y}, \theta)$ one can simplify the system and obtain the two-dimensional time-optimal control problem:

$$\begin{cases} \dot{\widetilde{x}} = 1 + u \cdot \widetilde{y} \\ \dot{\widetilde{y}} = -u \cdot \widetilde{x} \end{cases} \tag{2}$$

$$|u| \leq 1, \quad \widetilde{x}(t_0) = \widetilde{x}_0, \ \widetilde{y}(t_0) = \widetilde{y}_0, \ \widetilde{x}(t_1) = 0, \ \widetilde{y}(t_1) = 1. \tag{3}$$

The solution of this time-optimal control problem is rather complicated [1].

But it turns out that if the both choice of the direction of motion along the final circle is allowed (this corresponds to the time-optimal control problem (2) with *two* endpoints $(0, -1)$ and $(0, 1)$), then the solution of the time-optimal control problem is essentially simplified. In this paper, we describe the optimal synthesis and give examples of motion with various initial conditions.

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Control and stabilizability of rotating Timoshenko beam with the aid of the torque

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We consider the following system of equations depicting the rotating Timoshenko beam:

$$\varrho(x)\ddot{w}(x,t) - (K(x)(w'(x,t) + \xi(x,t))' = -\varrho(x)(x+r)\ddot{\theta}(t)$$
$$R(x)\ddot{\xi}(x,t) - (E(x)\xi'(x,t))' + K(x)(w'(x,t) + \xi(x,t)) = R(x)\ddot{\theta}(t).$$

The beam is clamped to a rotating disc propelled by an engine. We denote by r the radius of the disc and by $\theta = \theta(t)$, the rotation angle ($t \geqslant 0$). To a (uniform) cross section of the beam at a point x, $0 \leqslant x \leqslant 1$, we assign the following values: $E(x)$ – flexural rigidity, $K(x)$ – shear stiffness, $\varrho(x)$ – mass of the cross section, $R(x)$ – rotary inertia. E, K, ϱ and R are the real functions defined on $[0, 1]$ and bounded by two positive constants. Also, we assume they are twice differentiable with bounded derivatives. By $w(x, t)$ we understand the deflection of the center line of the beam and $\xi(x, t)$ is the rotation angle of the cross section area at the location x and at the time t.

Assuming there is no deformation at the clamped end, as a consequence of the energy balance law, we obtain the following boundary conditions:

$$w(0,t) = \xi(0,t) = 0$$
$$w'(1,t) + \xi(1,t) = \xi'(1,t) = 0.$$

for all $t \geqslant 0$. The control function u is the angular acceleration $(u(t) = \ddot{\theta}(t))$.

Let I_d denote the disc inertia. The control $\bar{u}$ with the aid of the torque is given by the equation

$$\begin{aligned}\bar{u}(t) &= I_d\ddot{\theta}(t) \\ &\quad + \int_0^1 \varrho(x)(x+r)\big(\ddot{w}(x,t) + (x+r)\ddot{\theta}(t)\big)dx \\ &\quad - \int_0^1 R(x)\big(\ddot{\xi}(x,t) - \ddot{\theta}(t)\big)dx.\end{aligned}$$

We will define and demonstrate solutions of various problems connected with controllability and stabilizability of the introduced model.

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Nanofluidic flows in the tubes and minimum entropy generation principle

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Flows of the suspensions of nanoparticles ($d \sim 1-500nm$ nanofluids) and microparticles ($d \sim 1-500mcm$ microfluids) through nano/microtubes, channels and other types of ducts are governed my the Navier-Stokes equations with velocity slip and temperature jump boundary conditions at the walls [1]. During the past decades numerous units and systems for mixing, purification, separation, heating and cooling of micro and nanofluids have been proposed for technical, electrical and biomedical applications [2]. In this paper the nanofluid flows through a circular microtube driven by the pressure drop at the ends of the tube, with heat exchange through the wall is studied. At some types of boundary conditions the analytical solution for the velocity v, pressure p and temperature T distributions can be obtained. The entropy production in the system can be then computed as [3]

$$\dot{S} = \frac{1}{V}\int_V \left(\frac{\nabla^2 T}{T^2} + ReEuEc\frac{v \cdot \nabla p}{T}\right) dV, \tag{1}$$

where Re, Eu and Ec are the Reynolds, Euler and Eckert numbers, V is the volume of the duct.

Numerical computations on (1) needs a large set of the material parameters. That allows finding out the optimal control and flow optimization as a relationships between the dimensionless combinations of the parameters. Some reasonable ways to minimize the dissipation in the micro heating or cooling system based on the nanosuspensions with certain properties are discussed.

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Asymptotic analysis and optimal decay ratio of damped slowly rotating Timoshenko beams

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A stability analysis was performed in the problem of a rotating Timoshenko beam whose movement is controlled by the angular acceleration of the driving motor into which the beam is rigidly clamped (cf. [1]). After introducing a damping effect with respect to a rotation angle of a cross section area of rotating Timoshenko beam model, we obtain [2] the following system of two dimensionless partial differential equations

$$\begin{cases} \ddot{w}(x,t) - w''(x,t) - \xi'(x,t) & = & -u(t)(r+x), \\ \ddot{\xi}(x,t) - \gamma^2\xi''(x,t) + w'(x,t) + \xi(x,t) + \nu^2\dot{\xi}(x,t) & = & u(t), \end{cases}$$

for $x \in (0,1)$ and $t > 0$, where ν is a damping constant, with boundary conditions

$$\begin{cases} w(0,t) = \xi(0,t) &= 0, \\ w'(1,t) + \xi(1,t) = \xi'(1,t) &= 0. \end{cases}$$

Next, we show some important spectral properties of operator connected with the system. Furthermore, we show the asymptotic stability of the system under certain assumptions on the physical parameter γ^2. We find the optimal damping coefficient, maximizing the stability margin of the system.

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Inverse spectral problem for the operators with non-local potential

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The main object under consideration in the presentation is the second derivative operator on a finite interval with zero boundary conditions perturbed by a self-adjoint integral operator with the degenerate kernel (non-local potential). The inverse problem, i. e., the reconstruction of the perturbation from the spectral data, is solved by means of the step-by-step procedure based on the n-interlacing property of the spectrum.

Stabilization of a nonlinear system with elastic plates

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Consider a mechanical system that consists of a rigid body and two elastic plates. The vibration of the plates is governed by the Kirchhoff equations (see, e.g., [1]):

$$\ddot{w}_1 + a_1^2 \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right)^2 w_1 = (x_1 + d_1)\dot{\omega}_2 - (x_2 + d_2)\dot{\omega}_1, \tag{1}$$

$$\ddot{w}_2 + a_2^2 \left(\frac{\partial^2}{\partial x_1'^2} + \frac{\partial^2}{\partial x_2'^2} \right)^2 w_2 = (x_1' + d_1')\dot{\omega}_2 - (x_2' + d_2')\dot{\omega}_1, \tag{2}$$

subject to the boundary conditions

$$w_j|_{\partial\Omega_j} = 0, \qquad \left.\frac{\partial^2 w_j}{\partial n^2}\right|_{\partial\Omega_j} = 0, \qquad j = 1, 2. \tag{3}$$

We exploit the angular momentum equations for the rigid body-carrier:

$$\dot{K} + \omega \times K = f, \tag{4}$$

where $K = I\omega + \rho_1 \int\limits_{\Omega_1} r_P \times v_P dx + \rho_2 \int\limits_{\Omega_2} r_K \times v_K dx'$, and use the Poisson kinematic equations:

$$\dot{g}_{i1} = \omega_3 g_{i2} - \omega_2 g_{i3},\ \dot{g}_{i2} = \omega_1 g_{i3} - \omega_3 g_{i1},\ \dot{g}_{i3} = \omega_2 g_{i1} - \omega_1 g_{i2}, \quad i = \overline{1,3}. \tag{5}$$

We rewrite the control system (1)–(5) as an abstract differential equation with respect to the state ξ in a suitable Hilbert space H and propose a feedback law $f = G\xi$, so that the closed-loop system takes the form

$$\frac{d}{dt}\xi(t) = F\xi(t), \qquad F = A + BG. \tag{6}$$

Here $A : D(A) \to H$ is unbounded nonlinear operator, $B : \mathbb{R}^3 \to H$ is bounded linear operator, and $G : H \to \mathbb{R}^3$.

We prove that the solution $\xi = 0$ of equation (6) is strongly stable in the sense of Lyapunov.

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