One problem of a control system design in the class C^1

Daria Andreieva, *Kharkiv, Ukraine*
Svetlana Ignatovich, *Kharkiv, Ukraine*

In the paper [1] a method for constructing a controllable system was proposed for the class of real-analytic vector fields. Let $\dot{x} = f(x)$ be a system of differential equations, where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}^n$, $n > 1$. The problem is to find a vector field $g(x)$ such that the system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

is completely controllable. The following theorem was proved in [1]:

Theorem 1. *Let a system $\dot{x} = f(x)$ be given. A vector field $g(x)$, for which the system (1) is controllable, exists if and only if $f \not\equiv 0$.*

The idea [1] of constructing a controllable system in the theorem is to straighten the given vector field $f(x)$, that is, to obtain $f = [0, \dots, 1]^T$ and, changing variables, to obtain a linear system that we already know how to make controllable. By performing the inverse change of variables, we obtain a controllable system in the initial coordinates.

In the report, this problem is considered for finitely differentiable vector fields, in particular, for vector fields of the class C^1. In this case a system design problem is related to linearizability problems studied in the paper [2]. As a result, we obtain the following theorem:

Theorem 2. *Let a system of differential equations*

$$\dot{x} = f(x) + g(x)u$$

be given, where the vector field $g(x)$ is constructed according to the method described above. If the vector field $f(x)$ is of the class C^2, then the vector field $g(x)$ is of the class C^1.

[1] Kawano, Y. Any dynamical system is fully accessible through one single actuator, and related problems / Y. Kawano, Ü. Kotta, C. H. Moog // International Journal of Robust and Nonlinear Control. – 2016. – Vol. 26, no. 8. – P. 1748-1754.

[2] Sklyar, G.M. On the extension of the Korobov's class of linearizable triangular systems by nonlinear control systems of the class C^1 / G.M. Sklyar, K.V. Sklyar, S.Yu. Ignatovich // Systems and Control Letters. – 2005. – Vol. 54. – P. 1097–1108.