Exact null controllability of time-delay systems as trigonometric moment problem

Pavel Barkhayev, *Kharkiv, Ukraine*
Rabah Rabah, *Nantes, France*
Grigory Sklyar, *Szczecin, Poland*

We analyze the relation between notions of exact null controllability and spectral controllability for a quite general class of linear time-delay systems of retarded type with distributed terms. One of the first results was obtained in [2] where two-dimensional systems of the form

$$\dot{x} = A_1 x(t-h) + A_0 x(t) + bu(t) \tag{1}$$

were considered. The authors proved that exact null controllability is equivalent to spectral controllability for such systems. In 1979 V. Marchenko [3] conjectured that this equivalence holds for much more general class of retarded systems. In 1984 Colonius [1] showed the equivalence property for systems (1) of arbitrary dimensions. His proof was based on the fact that spectrum controllability is equivalent to solvability of finite spectrum assignment problem. Later in [4] an explicit algebraic algorithm of computing a control function which steers any given initial function to the equilibrium position in finite time was given. This allowed to prove that spectral controllability implies null controllability for quite wide class of systems.

In this work we consider a more broad class of systems given by

$$\dot{z}(t) = A_1 z(t-1) + \int_{-1}^{0} [A_2(\theta)\dot{z}(t+\theta) + A_3(\theta)z(t+\theta)]\,\mathrm{d}\theta + Bu(t), \tag{2}$$

assuming that $\operatorname{rank}(A_1,\ B) = n$ and $\operatorname{supp} A_i(\theta) \subset [-1+\varepsilon, 0]$ for some $\varepsilon > 0$.

We study the problem of exact null controllability as an infinite vector moment problem assuming that spectral controllability holds. The approach we used is essentially based on the property of minimality of the operator's family of exponentials. This allows to construct steering controls and solve moment problem for each state of the model space.

[1] F. Colonius, *On approximate and exact null controllability of delay systems*, Systems Control Lett. 5 (1984), no. 3, 209–211.

[2] M.Q Jacobs, C. E. Langenhop, *Criteria for function space controllability of linear neutral systems*, SIAM J. Control Optimization 14 (1976), no. 6, 1009–1048.

[3] V. M. Marchenko, *On the controllability of zero function of time lag systems. (Russian)*, Problems Control Inform. Theory/Problemy Upravlen. Teor. Inform. 8 (1979), no. 5-6, 421–432.

[4] A. W. Olbrot, L. Pandolfi, *Null controllability of a class of functional-differential systems*, Internat. J. Control 47 (1988), no. 1, 193–208.

[5] R. Rabah, G.M. Sklyar, and P.Y. Barkhayev, *Exact null controllability, complete stabilizability and continuous final observability of neutral type systems*, Int.J.Appl.Math.Comp.Sci. 27(3) (2017), 489–499.