On synthesis problem for inherently nonlinear systems

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We study the controllability problem for a class of nonlinear systems of the form

$$\begin{cases} \dot{x}_1 = u, \quad |u(x)| \le d, \\ \dot{x}_i = x_{i-1}^{2k_{i-1}+1} + f_{i-1}(t,x,u), \quad i = 2,\ldots,n, \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control, $d > 0$ is a given number, $k_i = \frac{p_i}{q_i}$ ($p_i > 0$ is an integer, $q_i > 0$ is an odd integer), $f_i(t,x,u)$ ($i = 1,\ldots,n-1$) are continuous real-valued functions with $f_i(t,0,0) = 0$ for all $t \ge 0$.

We construct a class of bounded controls $u = u(x)$ such that for any initial point $x_0 \in U(0)$ the solution $x(t,x_0)$ of the corresponding closed-loop system is well-defined on the interval $[0, T(x_0)]$ and ends at 0 in a finite-time $T(x_0) < +\infty$, i.e. $\lim_{t \to T(x_0)} x(t,x_0) = 0$.

The class of smooth stabilizing controls for system (1) was proposed in [1]. The synthesis problem for the case when $f_i(t,x,u) = 0$ ($i = 1,\ldots,n-1$) and $k_i = 0$ ($i = 1,\ldots,n-2$), $k_{n-1} > 0$ was solved in [2]. The approach which was proposed in [2] for constructing finite-time stabilizers is based on the controllability function method [3]. Under some additional growth conditions imposed on functions $f_i(t,x,u)$ we develop this approach to construct a class of bounded finite-time stabilizing controls $u = u(x)$ for system (1). To this end, we construct a class of controllability functions $\Theta(x)$ such that the inequality $\dot{\Theta}(x) \le -\beta \Theta^{1-\frac{1}{\alpha}}(x)$ holds for some $\alpha \ge 1$, $\beta > 0$. The former inequality guarantees that any trajectory of the closed-loop system starting in $U(0)$ hits the origin in some finite time $T(x_0)$.

[1] Bebiya M.O. and Korobov V.I., *On Stabilization Problem for Nonlinear Systems with Power Principal Part* // Journal of Mathematical Physics, Analysis, Geometry, 2016, Vol. 12, No. 2, 113–133.

[2] Bebiya M.O., *Global synthesis of bounded controls for systems with power nonlinearity* // Visnyk of V.N. Karazin Kharkiv National University, Ser. Mathematics, Applied Mathematics and Mechanics, 2015, Vol. 81, 36–51.

[3] Korobov V.I., *The method of controllability function*, R&C Dynamics, M.-Izhevsk, 2007 (in Russian).