Constructive methods of investigation of the differential-algebraic Cauchy problem with degenerate pulse action

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We investigate the problem of the determination of constructive conditions for the existence of solution [1]

$$z(t) \in \mathbb{C}^1\{[a;b] \setminus \{\tau_i\}_I\}$$

of the linear differential-algebraic equation [2,3]

$$A(t)z'(t) = B(t)z(t) + f(t), \quad t \neq \tau_i, \ i = 1, \ 2, \ \dots \ , \ p \tag{1}$$

with the impulse action [1,4]

$$\Delta z(\tau_i) = \mathcal{S}_i \ z(\tau_i - 0) + a_i, \quad \mathcal{S}_i \in \mathbb{R}^{n\times n}, \quad \tau_i \in [a,b], \quad a_i \in \mathbb{R}^n. \tag{2}$$

The matrices

$$A(t), \ B(t) \in \mathbb{C}_{k\times n}[a,b] := \mathbb{C}[a,b] \otimes \mathbb{R}^{m\times n}, \quad m \neq n$$

and the vector function $f(t) \in \mathbb{C}[a,b]$ are assumed to be continuous on the segment $[a,b]$. Provided

$$\det(I_n + \mathcal{S}_i) = 0, \ i = 1, \ 2, \ \dots \ , \ p$$

for the principal solution matrix $X(t)$ of the linear differential-algebraic equation (1) holds degenerate case [5,6].

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