About an approximate solution of matrix differential-algebraic boundary-value problems with a least-squares method

Sergey Chuiko, *Slavyansk, Ukraine*
Olga Nesmelova, *Slavyansk, Ukraine*
Marina Dzuba, *Slavyansk, Ukraine*

We investigate the problem of the determination of conditions for the existence of solution [1]

$$Z(t) \in \mathbb{C}^1_{\alpha\times\beta}[a;b] := \mathbb{C}^1[a;b] \otimes \mathbb{R}^{\alpha\times\beta}$$

of the matrix differential-algebraic equation [2,3,4]

$$\mathcal{A}Z'(t) = \mathcal{B}Z(t) + F(t), \tag{1}$$

that satisfy the boundary condition

$$\mathcal{L}Z(\cdot) = \mathfrak{A}, \quad \mathfrak{A} \in \mathbb{R}^{\mu\times\nu} \tag{2}$$

and the construction of this solution. Here,

$$\mathcal{A}Z'(t): \ \mathbb{C}^1_{\alpha\times\beta}[a,b] \to \mathbb{C}_{\gamma\times\delta}[a,b], \ \ \mathcal{B}Z(t): \ \mathbb{C}^1_{\alpha\times\beta}[a,b] \to \mathbb{C}^1_{\gamma\times\delta}[a,b]$$

is a matrix operator, which ensures, by definition, the equality [5,6]

$$\mathcal{A}(\zeta'(t)\Xi_1 + \xi'(t)\Xi_2)(t) = \zeta'(t)\mathcal{A}(\Xi_1)(t) + \xi'(t)\mathcal{A}(\Xi_2)(t),$$

$$\mathcal{B}(\zeta(t)\Xi_1 + \xi(t)\Xi_2)(t) = \zeta(t)\mathcal{B}(\Xi_1)(t) + \xi(t)\mathcal{B}(\Xi_2)(t)$$

for any functions $\zeta(t), \xi(t) \in \mathbb{C}^1[a,b]$ and any constant matrices Ξ_1, Ξ_2.

[1] Boichuk A. A., Samoilenko A. M. Generalized Inverse Operators and Fredholm Boundary-value Problems 2-nd edition, Walter de Gruyter GmbH & Co KG, 2016: 1-298.

[2] Campbell S.L. Singular Systems of differential equations, Pitman Advanced Publishing Program, San Francisco-London-Melbourne, 1980: 1-178.

[3] Chuiko S. M. On the lowering of the order of the matrix differential-algebraic system (in Russian) // Ukrainian math. bulletin. – 2018. – 15. – № 1. – pp. 1 – 17.

[4] Boichuk A. A., Krivosheya S. A. A critical periodic boundary value problem for a matrix riccati equation // Differential Equations. – 2001. – 37. – № 4. – pp. 464 – 471.

[5] Chuiko S. M. The Green's operator of a generalized matrix linear differential-algebraic boundary value problem // Siberian Mathematical Journal. – 2015. – 56. – № 4. – pp. 752 – 760.

[6] Chuiko S. M., Nesmelova O. V., Dzuba M. V. Least-squares method in the theory of matrix differential-algebraic boundary-value problems // Ukr. Mat. Zh. – 2018. – 70. – № 2. – pp. 280 – 292.