Constructive methods of investigation of the differential-algebraic Cauchy problem

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We investigate the problem of the determination of constructive conditions for the existence of solution $z(t) \in \mathbb{C}^1[a,b]$ of the linear differential-algebraic equation [1,2,3]

$$A(t)z'(t) = B(t)z(t) + f(t). \tag{1}$$

The matrices

$$A(t),\ B(t) \in \mathbb{C}_{m\times n}[a,b] := \mathbb{C}[a,b] \otimes \mathbb{R}^{m\times n}, \quad m \neq n$$

and the vector function $f(t) \in \mathbb{C}[a,b]$ are assumed to be continuous on the segment $[a,b]$.

Found solvability conditions and construction of the generalized Green operator of the Cauchy problem for a linear differential-algebraic system (1). Found sufficient conditions for reducibility generalized matrix differential-algebraic equation (1) to a sequence of systems combining differential and algebraic equations. An original classification is proposed, as well as a unified scheme for constructing solutions of differential-algebraic equations (1).

The method for construction of solvability conditions and construction of the generalized Green operator for linear differential-algebraic equation (1) can be generalized to boundary value problem for the matrix differential-algebraic equations in various critical and noncritical cases [4,5,6].

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