

The Stieltjes matrix moment problem and associated positive symmetric operators

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A sequence of $m \times m$ matrices $(s_j)_{j=0}^{\infty}$ is called \mathbb{R} - positive (\mathbb{R}_+ - positive) if, for all $l > 0$, the Hankel matrices $H_1^{(l)} = (s_{j+k})_{j,k=0}^l$ are positive (the Hankel matrices $H_1^{(l)}$ and $H_2^{(l)} = (s_{j+k+1})_{j,k=0}^l$ are positive, respectively).

Let the sequence $(s_j)_{j=0}^{\infty}$ be \mathbb{R} - positive. Then nonnegative matrix measures σ are called *solutions* to the Hamburger matrix moment problem if

$$s_j = \int_{-\infty}^{\infty} t^j \sigma(dt), \quad j \geq 0.$$

Let the sequence $(s_j)_{j=0}^{\infty}$ be \mathbb{R}_+ - positive. Then nonnegative matrix measures σ are called *solutions* to the Stieltjes matrix moment problem if

$$s_j = \int_0^{\infty} t^j \sigma(dt), \quad j \geq 0.$$

The ranks of the radii of the limit Weyl discs are the geometric measure of degeneracy of the solution set to the Hamburger moment problem. The deficiency numbers of the associated symmetric operator are the operator measure of degeneracy of the solution set. Note that the geometric and operator measures of degeneracy are equal.

The ranks of the limit matrix Weyl intervals (see [1]) are the geometric measure of degeneracy of the solution set to the Stieltjes moment problem. However, the operator measure of degeneracy of the solution set to the Stieltjes matrix moment problem has remained uninvestigated. We introduce the operator measure of degeneracy of the solution set to the Stieltjes matrix moment problem in terms of the deficiency subspaces of a pair of positive symmetric operators, which is a novel approach. A relation between the operator and geometric measures of degeneracy of the solution set to the Stieltjes matrix moment problem is established. As a corollary, some results for the Stieltjes matrix moment problem are proved (see [2]).

- [1] Dyukarev Yu. M. Indeterminacy criteria for the Stieltjes matrix moment problem. // Mathematical Notes. – 2004. – 75(1). – pp. 66-82.
- [2] Dyukarev Yu. M. Geometric and operator measures of degeneracy for the set of solutions to the Stieltjes matrix moment problem. // Sb. Math. – 2016. – 207(4). – pp. 519 - 536.