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Linear operator-differential equation with generalized quasipolinomial on the right-hand side

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We consider the Cauchy problem

$$u'(z) = Au(z) + e^{\gamma z} f(z), \quad z \in \mathbb{C}. \tag{1}$$

$$u(0) = b \in D(A), \tag{2}$$

where A is a closed operator on a complex Banach space X with a domain $D(A)$ ($D(A)$ is not necessarily dense in X), $\gamma \in \mathbb{C}$ and $f : \mathbb{C} \to X$ is an entire vector-valued function of zero exponential type.

Theorem 1. *If γ is a regular point of the operator A, then Equation (1) has the following unique solution of the form $e^{\gamma z} v(z)$, where $v(z)$ is an entire vector-valued function of zero exponential type,*

$$u(z) = -e^{\gamma z} \sum_{n=0}^{\infty} (A - \gamma I)^{-(n+1)} f^{(n)}(z).$$

Thus, the Cauchy problem (1), (2) has a solution of the above form if and only if $b = -\sum_{n=0}^{\infty} (A - \gamma I)^{-(n+1)} f^{(n)}(0)$.

Now, we assume that γ is an isolated point of the spectrum of A. We introduce the spectral projection P_γ corresponding to γ and expand the operator $A = A_\gamma \dot{+} \widetilde{A}_\gamma$ with respect to the direct sum $X = X_\gamma \dot{+} \widetilde{X}_\gamma$, $X_\gamma = P_\gamma(X)$, $\widetilde{X}_\gamma = (I - P_\gamma)(X)$.

Theorem 2. *If the operator $A - \gamma I$ is not quasinilpotent, then the Cauchy problem (1), (2) has a solution of the form $e^{\gamma z} v(z)$, where $v(z)$ is an entire vector-valued function of zero exponential type if and only if*

$$(I - P_\gamma) b = -\sum_{n=0}^{\infty} (\widetilde{A}_\gamma - \gamma I)^{-(n+1)} (I - P_\gamma) f^{(n)}(0).$$

Moreover, such a solution is unique and admits the representation

$$u(z) = e^{zA_\gamma} P_\gamma b + \int_0^z e^{\gamma \zeta} e^{(z-\zeta)A_\gamma} P_\gamma f(\zeta) d\zeta -$$

$$-e^{\gamma z} \sum_{n=0}^{\infty} (\widetilde{A}_\gamma - \gamma I)^{-(n+1)} (I - P_\gamma) f^{(n)}(z).$$