Approximate solutions of the Boltzmann equation with infinitely many modes

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We consider the nonlinear kinetic Boltzmann equation in case of a model of hard spheres [1]. We construct an approximate solution in the form

$$f(t,x,V)=\sum_{i=1}^{\infty}\varphi_i(t,x)M_i(V), \tag{1}$$

where $\varphi_i(t,x)$ are smooth, nonnegative and bounded on R^4 functions. The exact solutions $M_i(V)$ are global Maxwellians:

$$M_i(V)=\rho_i\left(\frac{\beta_i}{\pi}\right)^{3/2}e^{-\beta_i\left(V-\overline{V}_i\right)^2}.$$

We use the uniform-integral error:

$$\Delta=\sup_{(t,x)\in\mathbb{R}^4}\int_{\mathbb{R}^3}dV\Big|D(f)-Q(f,f)\Big|.$$

Theorem 1. *Let the coefficient functions $\varphi_i(t,x)$ in the distribution (1) be such that the functional series:*

$$\sum_{i=1}^{\infty}\varphi_iM_i,\quad \sum_{i=1}^{\infty}|V|\varphi_iM_i,\quad \sum_{i=1}^{\infty}M_i\left|\frac{\partial\varphi_i}{\partial t}\right|,\quad \sum_{i=1}^{\infty}M_i|V|\left|\frac{\partial\varphi_i}{\partial x}\right|$$

converge uniformly in the whole space R^4.

Then there exists such a quantity Δ', that $\Delta\leqslant\Delta'$ and $\lim\limits_{\beta_i\to+\infty}\Delta'$ is equal to:

$$\sum_{i=1}^{\infty}\rho_i\sup_{(t,x)\in\mathbb{R}^4}\left|\frac{\partial\varphi_i}{\partial t}+\left(\overline{V}_i,\frac{\partial\varphi_i}{\partial x}\right)\right|+2\pi d^2\sum_{\substack{i,j=1\\ i\neq j}}^{\infty}\rho_i\rho_j\Big|\overline{V}_i-\overline{V}_j\Big|\sup_{(t,x)\in\mathbb{R}^4}(\varphi_i\varphi_j).$$

The quantity Δ will be arbitrary small in the case, if $\varphi_i(t,x)=C_i(x-\overline{V}_it)$ or $\varphi_i(t,x)=E_i\left(\left[x,\overline{V}_i\right]\right)$ and with a special selection of hydrodynamic parameters.

[1] Cercignani C. The Boltzman Equation and its Applications, Springer, New York, 1988: 455.

[2] Gordevskyy V. D., Hukalov O. O. Approximate solutions of the Boltzmann equation with infinitely many modes // Ukr. Mat. Zh. – 2017. – 69(3). - pp. 311-323.(Ukrainian)