On bioheat equation and its modifications

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Mathematical modeling of the optical and infrared heating of biological tissues is based on the Pennes bioheat balance equation [1] and its modifications, the single-phase lag (SFL) [2] and dual-phase lag (DFL) [3] models. The Pennes bioheat equation is

$$\rho c \frac{\partial T}{\partial t} = div\,(k\nabla T) + q_{met} = q_h + \rho_b c_b w_b (T_b - T), \tag{1}$$

where T is the temperature, ρ, c, k are the density, specific heat and thermal conductivity of the tissue, the subscript b relates to the blood, w_b is the blood perfusion rate, q_{met} and q_h are metabolic and photostimulated heats.

The SFL model accounts for the time delay τ_q between the heat flux q and the temperature gradient ∇T that give the equation

$$\rho c \tau_q \frac{\partial^2 T}{\partial t^2} + (\rho c + \rho_b c_b w_b \tau_q) \frac{\partial T}{\partial t} = div\,(k\nabla T) + q_{met} + q_h + \rho_b c_b w_b T_b. \tag{2}$$

The DTL model accounts for two time lags τ_q and τ_T and has the form similar to (2). As it was shown, the SFL and DFL models are thermodynamically inconsistent, while the Guyer-Krumhansl equation as an example of the non-Fourier heat conduction law is thermodynamically correct.

In this paper the 1D solutions of the models (1)-(2) and the Guyer-Krumhansl equation for the surface heating of human skin are considered. The computational results are compared to the measured curves $T(t)$. It is shown, the Guyer-Krumhansl equation gives the best correspondance between the computational and measured curves for both heating and relaxation precesses.

[1] Xu F. , Lu T. J., Seffen K. A. Biothermomechanics of skin tissues. // J. Mech. Phys. Solids. – 2008. – 56(4). – pp. 1852 - 1884.

[2] Ciesielski M., Mochnacki B. , Piasecka-Belkhayat A. Analysis of temperature distribution in the heated skin tissue under the assumption of thermal parameters uncertainty. // 40th Solid Mechanics Conference. Warsaw, IPPT. – 2016. – P048.

[3] Nóbrega S. , Coelho P. J. A parametric study of thermal therapy of skin tissue. // J. Thermal Biol. – 2017. – 63(2). – pp. 92 - 103.