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Feedback synthesis for motion of a material point with allowance for friction

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Let us consider the feedback synthesis for motion of material point with allowance for friction:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ p(t, x_1, x_2)x_2 + u \end{pmatrix}. \tag{1}$$

Here $t \geq 0$, $(x_1, x_2) \in \mathbb{R}^2$ is a state; u is a scalar control, $|u| \leq 1$; $p(t, x_1, x_2)$ is *unknown* nonlinear viscous friction, $p_1 \leq p(t, x_1, x_2) \leq p_2$. The approach presented in the talk is based on the controllability function method proposed by V.I. Korobov in 1979. In [1] a control $u(x)$ solving the feedback synthesis problem for system (1) without friction was given. It satisfies two conditions:
1) $|u(x)| \leq 1$; 2) the trajectory $x(t)$ starting from an initial point $x(0) = x_0 \in \mathbb{R}^2$ of the closed system $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ u(x) \end{pmatrix}$ ends at the origin at a finite time $T > 0$, and, in addition, the time T is equal to $\Theta(x_0)$ for any $x_0 \in \mathbb{R}^2$.

The *main goal* of the research is to find friction limits such that a control steering the system without friction to the origin also steers the system with friction to the same target.

Theorem 1. *Let $a_1 < -4.5$, $0 < \gamma_1 < 1$, $\gamma_2 > 1$, $c > 0$. The controllability function $\Theta = \Theta(x_1, x_2)$ is defined for $x \neq 0$ as a unique positive solution of the equation*

$$\frac{(4 + a_1)\Theta^4}{a_1(3 + a_1)} - a_1 x_1^2 + 4\Theta x_1 x_2 + \Theta^2 x_2^2 = 0. \tag{2}$$

Let $Q = \{(x_1, x_2) \mid \Theta(x_1, x_2) \leq c\}$, $\quad u(x) = \dfrac{a_1 x_1}{\Theta^2(x_1, x_2)} - \dfrac{3x_1}{\Theta(x_1, x_2)}$,

$$p_1^0 = \max\{(1 - \gamma_1)\tilde{p}_1^0; (1 - \gamma_2)\tilde{p}_2^0\}, \qquad p_2^0 = \min\{(1 - \gamma_1)\tilde{p}_2^0; (1 - \gamma_2)\tilde{p}_1^0\},$$

$$\tilde{p}_1^0 = \frac{3 + a_1 - \sqrt{a_1(a_1 + 4)}}{c}, \qquad \tilde{p}_2^0 = \frac{3 + a_1 + \sqrt{a_1(a_1 + 4)}}{c}.$$

Then, for all $p_1 \leq p(t, x_1, x_2) \leq p_2$ such that $[p_1; p_2] \subset (p_1^0; p_2^0)$, the trajectory of the closed system starting from an initial point $x(0) = x_0 \in Q$ ends at the point $x(T) = 0$ at a finite time $T = T(x_0, p_1, p_2)$ under the estimate $\Theta(x_0)/\gamma_2 \leq T(x_0, p_1, p_2) \leq \Theta(x_0)/\gamma_1$.

Furthermore we analyze the envelope for one-parametric family (2) at $\Theta = 1$ for the system (1) with $p(t, x_1, x_2) = 0$. It is close to the curve that describes all points from which we may steer to the origin due to the Pontryagin maximum principle for the time $t = 1$.

[1] Choque Rivero A. E., Korobov V. I., Skoryk V. A. The controllability function as the time of motion 1.// Jour. Math. Phys. Anal. Geom. – 2004. – 11(2). – pp. 208 - 225.