On one class of non-dissipative operators

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The study of the basicity of systems of functions, as a rule, is based on the study of some properties of linear operators. The study of the so-called class of quasi-exponentials provokes special interest, it is started by B. S. Pavlov and then developed and continued by S. V. Hruščev, N. K. Nikolsky, B. S. Pavlov. An approach suggested by G. M. Gubreyev is an important method of studying problems of basicity in this realm of analysis. He succeeded in harmonic combination of deep problems of spectral analysis of non-selfadjoint operators and delicate analytical results of the theory of functions. This work is a development of ideas of the paper by G. M. Gubreyev and V. N. Levchuk in which the study of Dunkl kernels is based on the analysis of a non-selfadjoint operator with two-dimensional imaginary component. (The function $d_\alpha(\lambda) = 2^\alpha \Gamma(\alpha+1)\lambda^{-\alpha}\left(J_\alpha(\lambda) + iJ_{\alpha+1}(\lambda)\right)$ is said to be a Dunkl kernel, where $J_\alpha(\lambda)$ is a Bessel function.) In contrast to, here the power dependence of the weight function is not supposed. This work is dedicated to the study of one class of Volterra non-dissipative operators and to the construction of model representations for them. It turns out that many statements from are general and can be obtained for “arbitrary” weight functions $\varphi(x)$. General properties of the operator B are studied ant its characteristic function is calculated. Calculation of this characteristic function is based on the solution of the equation of the second order which depending on the choice of the weight $\varphi(x)$ turns into either a Bessel equation, a Mathieu equation, or a Lamé equation. Similarity of the studied non-dissipative operator to the operator of integration in the space of quadratically summed functions on $[-a, a]$ is proved. A functional model of a non-dissipative operator in the L. de Branges space is listed, and it is shown that in the special case, when $\varphi(x) = x^\nu$, the Dunkl kernels “coincide” with $E(\lambda)$.