On exact controllability and complete stabilizability of linear systems in Hilbert spaces

Rabah Rabah, *Nantes, France*

We consider linear systems in the general form

$$\dot{x} = \mathcal{A}x + \mathcal{B}u, \tag{1}$$

where the state $x(t)$ and the control $u(t)$ take values in Hilbert spaces X and U. A is a linear operator, infinitesimal generator of a C_0-semigroup $S(t)$, B is linear bounded operator. By exact (null) controllability we mean controllability from any state to any state (or zero state). By complete stabilizability we mean exponential stabilizability with arbitrary decay rate or, sometimes pole assignment, by linear state feedback $u = \mathcal{F}x$.

It is well known (cf. for example [2]) that in an finite dimensional setting exact controllability (said complete controllability) is a necessary and sufficient condition for complete stabilizability or more precisely arbitrary pole assignment. The situation is more complicated in infinite dimensional spaces.

We recall some classical results concerning the relation between exact controllability and complete stabilizability.

The first important result in this context was given by Slemrod [1]: if $S(t)$ is a group, exact controllability implies complete stabilizability. The converse, for a group, was proved by Zabczyk [3]. The result was generalized and precized by several authors for the case of a bounded operator A, for the case of a semigroup $S(t)$ (not a group) and for some classes of systems, governed by partial differential equations or functional-differential equations (with delays).

We discuss more precisely the relations between exact null controllability and complete stabilizability. For the system (1), exact null controllability implies complete stabilizability, but the converse is not true. We give more recent results on functional-differential systems of neutral type, which may be represented in the form of system (1), and described by the equation:

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)\,\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)\,\mathrm{d}\theta + Bu.$$

[1] M. Slemrod. A note on complete controllability and stabilizability for linear control systems in Hilbert space.// SIAM J. Control. – 1974. – 12(3). – pp. 500 - 508.

[2] W. M. Wonham. Linear multivariable control: A geometric approach, Springer, New York, 3d ed., 1985.

[3] J. Zabczyk. Complete stabilizability implies exact controllability.// Seminarul Ecuati Functionale. – 1976. – 38. – pp. 1 - 7.