Stopping of oscillations of controlled elliptic pendulum

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This work deals with the feedback synthesis problem for controlled elliptic pendulum. The main idea of the article is to find the control that will steer the initial point to the origin. The equations describing the motion of a controlled elliptic pendulum were constructed. They are provided below.

$$\begin{cases} \ddot{y} = -\dfrac{glm_2\varphi + lv_1 - v_2}{lm_1}, \\ \ddot{\varphi} = \dfrac{-\varphi glm_2(m_1+m_2) - lm_2v_1 + (m_1+m_2)v_2}{l^2m_1m_2}, \end{cases} \tag{1}$$

where $y(t)$ is the horizontal axis deflection and $\varphi(t)$ is the the deviation from the bottom stability stance. The method of controlability function invented by V.I. Korobov was used. Firstly, using the next change of variables:

$$z_1 = \varphi, z_2 = \dot{\varphi}, z_3 = y, z_4 = \dot{y} \tag{2}$$

we've transformed the 2-dimensional system into the linear 4-dimensional system. After that, we have transformed it to the canonical system using the linear change of variables $x = Lz$. Using abovementioned method, bounded control that stops the oscillation of this mechanical system was constructed. The controllabiblity function $\Theta(x)$ is defined as a unique positive solution of the equation

$$2a_0\Theta^4 = 36x_1^2 + 24\Theta x_1x_2 + 6\Theta^2x_2^2 + 36x_3^2 + 24\Theta x_3x_4 + 6\Theta^2x_4^2, \tag{3}$$

for some $a_0 > 0$. The next equality should be used to find the necessary control.

$$v(x) = \begin{pmatrix} v_1(x) \\ v_2(x) \end{pmatrix} = \begin{pmatrix} -\dfrac{6x_1}{\Theta^2(x)} - \dfrac{3x_2}{\Theta(x)} \\ gx_1 + x_3\dfrac{g(m_1+m_2)}{lm_1} - \dfrac{6x_3}{\Theta^2(x)} - \dfrac{3x_4}{\Theta(x)} \end{pmatrix} \tag{4}$$

This control steers an arbitrary initial point of a certain neighborhood of the origin of the coordinates to the origin in a finite time. Graphics of the trajectory and control on the trajectory, which begin from the chosen initial point, are presented.

[1] Korobov V. I. The method of controllability function (Russian), R&C Dynamics, M.-Izhevsk, 2007: 1-576.