Solutions to nonlinear systems of reaction-diffusion equations/ ODEs with delay

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We are interested in a class of systems of non-linear partial differential equations/ODEs with different types of bounded time delays. To describe systems, we remind the notation which is usual in the theory of delay equations. Considering the maximal delay $h > 0$, for a function $v(t), t \in [a-h, b] \subset \mathbb{R}, b > a$, we denote the history segment $v_t = v_t(\theta) \equiv v(t+\theta), \theta \in [-h, 0], t \in [a, b]$.

The general form of a delay system under consideration is

$$\frac{d}{dt}u(t) + Au(t) = F(u_t), \tag{1}$$

In (1), A is an unbounded linear operator in a Banach space X, $F : C \equiv C([-h, 0]; X) \to X$ is a nonlinear (delay) map. The form of F depends on particular applied problems. Initial conditions, in general, are

$$u|_{[-h,0]} = \varphi \in C \equiv C([-h, 0]; X). \tag{2}$$

For particular cases, the set of initial functions φ could be a carefully choosen subset (not necessarily linear) of the space C.

We are interested in reaction-diffusion systems in bounded domains with different types of delay in reaction terms. Particular interest is in the case of presence of discrete state-dependent delays. This type of delay is the most relevant to real-world applications and most difficult from mathematical point of view. For a survey on the ODE theory see [1]. The well-posedness in the sense of Hadamard and long time asymptotic behaviour of different types of solutions to (1)-(2) are studied (see e.g. [2, 3, 4]).

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[2] A. V. Rezounenko, A condition on delay for differential equations with discrete state-dependent delay // Journal of Mathematical Analysis and Applications. – 2012. – 385, – pp.506–516.

[3] A.V. Rezounenko, P. Zagalak, Non-local PDEs with discrete state-dependent delays: well-posedness in a metric space// Discrete and Continuous Dynamical Systems-Series A. – 2013. – 33(2). – pp. 819-835.

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