The integrable nonlocal nonlinear Schrödinger equation: Riemann-Hilbert approach and long-time asymptotics

Dmitry Shepelsky, *Kharkiv, Ukraine*
Yan Rybalko, *Kharkiv, Ukraine*

We study the initial value problem for the integrable nonlocal nonlinear Schrödinger (NNLS) equation

$$iq_t(x,t) + q_{xx}(x,t) + 2q^2(x,t)\bar{q}(-x,t) = 0$$

with decaying (as $x \to \pm\infty$) boundary conditions as well as with the step-like boundary conditions: $q(x,0) \to 0$ as $x \to -\infty$ and $q(x,0) \to A$ as $x \to -\infty$, where $A \neq 0$.

The main aim is to describe the long-time ($t \to +\infty$) behavior of the solution of these problems. To do this, we adapt the nonlinear steepest-decent method to the study of the Riemann-Hilbert problem associated with the NNLS equation. In the case of decaying initial data, our main result is that, in contrast to the local NLS equation, where the main asymptotic term (in the solitonless case) decays to 0 as $O(t^{-1/2})$ along any ray $x/t = const$, the power decay rate in the case of the NNLS depends, in general, on x/t, and can be expressed in terms of the spectral functions associated with the initial data.

In the case of the step-like boundary conditions, the asymptotics turns to be different in different sectors of the (x,t) plane. Particularly, in the right-most sector, the main asymptotic terms is a constant depending on the ratio x/t whereas the second term decays, as in the previous case, with the power decay rate depending on x/t.