Vanishing of solution of the model representative of NPE

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The theory of quasilinear parabolic equations has been developed since the 50-s of the 19th century. The properties of these equations differ greatly from those of linear equations. These differences were revealed in the scientific papers of the mathematicians: Barenblatt G.I., Oleinic O.A., Kalashnikov A.S., Zhou Yu Lin and others. Specific properties of NE (inertia, strongweaked localization of solutions' supports, extinction...) were studied by J.I. Diaz, L. Veron, A.E. Shishkov, B. Helffer, Y. Belaud, D. Andreucci and others. The most important aspect of such investigations is the description of structural conditions affecting the appearance and disappearance of various non-linear phenomena. Our investigation deals with nonlinear parabolic equation with degenerating absorption potential h(t), the presence of which play the important role in the study of the above mentioned properties.

So, we study Cauchy-Neumann problem for the next type of a quasilinear parabolic equation with the model representative:

$$u_t - \Delta u + h(t)|u|^{q-1}u = 0 \quad \text{in}\, \Omega \times (0,T) \tag{1}$$

$$\frac{\partial u}{\partial n}|_{\partial\Omega\times[0,T]} = 0 \tag{2}$$

$$u(x,0) = u_0(x), \qquad \mathbb{R}^N \backslash \{\operatorname{supp} u_0\} \neq \emptyset, \{\operatorname{supp} u_0\} \subset \{|x| < 1\} \tag{3}$$

Here $0 < q < 1$, the initial function $u_0(x) \in L_2(\Omega)$, $\Omega \subset \mathbb{R}^N (N \geqslant 1)$ be a bounded domain with C^1 - boundary. Assume that h(t) is a continuous, nonnegative, nondecreasing function, such that $h(0) = 0$. Let $h(t) = exp(-\frac{\omega(t)}{t})$, where $\omega(t)$ satisfies following technical conditions: (A) $\omega(t) > 0 \ \ \forall\, t > 0$, (B) $\omega(0) = 0$, (C) $\frac{t\,\omega'(t)}{\omega(t)} \leq 1 - \delta \ \ \forall\, t \in (0, t_0)$, $t_0 > 0$, $0 < \delta < 1$.

Theorem 1. *Let be an arbitrary function from $L_2(\Omega)$, $\omega(t)$ is continuous and nondecreasing function satisfy assumptions (A)(B)(C), then an arbitrary solution $u(x,t)$ of the problem (1)(2)(3) vanishes on Ω in some finite time $T < \infty$.*

To prove that, we use local energy method, which deals with norms of solutions $u(x,t)$ only and, therefore, may applied for higher order equations too.

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