One optimal control problem for an unmanned aerial vehicle

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The papers [1, 2] deal with one problem of minimizing the time for a kinematic model of unmanned aerial vehicle moving at a constant altitude. From a kinematic point of view, an UAV flying at a constant altitude is determined by standard Dubins equations [3]. Under additional speed constraints, the flight model of a drone is described by the following system of differential equations:

$$\dot{x} = \cos\theta, \quad \dot{y} = \sin\theta, \quad \dot{\theta} = u, \tag{1}$$

with $(x, y, \theta) \in \mathbb{R}^2 \times \mathbb{S}^1$ (where $(x, y) \in \mathbb{R}^2$ is UAV coordinates in the plane of constant height, θ is the angle of deviation from the course) and the control $u \in [-1, 1]$. In [1, 2] this (and more general) time-optimal problem was considered with the following final conditions: the UAV steers to the circle of radius 1 centered at the origin and then moves along it clockwise. Due to such final conditions, choosing a new basis $(\widetilde{x}, \widetilde{y}, \theta)$ one can simplify the system and obtain the two-dimensional time-optimal control problem:

$$\begin{cases} \dot{\widetilde{x}} = 1 + u \cdot \widetilde{y} \\ \dot{\widetilde{y}} = -u \cdot \widetilde{x} \end{cases} \tag{2}$$

$$|u| \leq 1, \quad \widetilde{x}(t_0) = \widetilde{x}_0, \ \widetilde{y}(t_0) = \widetilde{y}_0, \ \widetilde{x}(t_1) = 0, \ \widetilde{y}(t_1) = 1. \tag{3}$$

The solution of this time-optimal control problem is rather complicated [1].

But it turns out that if the both choice of the direction of motion along the final circle is allowed (this corresponds to the time-optimal control problem (2) with *two* endpoints $(0, -1)$ and $(0, 1)$), then the solution of the time-optimal control problem is essentially simplified. In this paper, we describe the optimal synthesis and give examples of motion with various initial conditions.

[1] Maillot T., Boscain U., Gauthier J.-P., Serres U. Lyapunov and minimum-time path planning for drones // J. Dyn. Control Syst. – 2015. – V. 21. – P. 47-80.

[2] Lagache M.-A., Serres U., Andrieu V. Minimal time synthesis for a kinematic drone model // Math. Control and Related Fields. – 2017. – 7(2). – P. 259-288.

[3] Agrachev A. A., Sachkov Yu. L. Control Theory from the Geometric Viewpoint. – Springer-Verlag Berlin Heidelberg, 2004. – 412 p.