Control and stabilizability of rotating Timoshenko beam with the aid of the torque

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We consider the following system of equations depicting the rotating Timoshenko beam:

$$\varrho(x)\ddot{w}(x,t) - (K(x)(w'(x,t) + \xi(x,t))' = -\varrho(x)(x+r)\ddot{\theta}(t)$$
$$R(x)\ddot{\xi}(x,t) - (E(x)\xi'(x,t))' + K(x)(w'(x,t) + \xi(x,t)) = R(x)\ddot{\theta}(t).$$

The beam is clamped to a rotating disc propelled by an engine. We denote by r the radius of the disc and by $\theta = \theta(t)$, the rotation angle ($t \geqslant 0$). To a (uniform) cross section of the beam at a point x, $0 \leqslant x \leqslant 1$, we assign the following values: $E(x)$ – flexural rigidity, $K(x)$ – shear stiffness, $\varrho(x)$ – mass of the cross section, $R(x)$ – rotary inertia. E, K, ϱ and R are the real functions defined on $[0,1]$ and bounded by two positive constants. Also, we assume they are twice differentiable with bounded derivatives. By $w(x,t)$ we understand the deflection of the center line of the beam and $\xi(x,t)$ is the rotation angle of the cross section area at the location x and at the time t.

Assuming there is no deformation at the clamped end, as a consequence of the energy balance law, we obtain the following boundary conditions:

$$w(0,t) = \xi(0,t) = 0$$
$$w'(1,t) + \xi(1,t) = \xi'(1,t) = 0.$$

for all $t \geqslant 0$. The control function u is the angular acceleration ($u(t) = \ddot{\theta}(t)$).

Let I_d denote the disc inertia. The control $\bar{u}$ with the aid of the torque is given by the equation

$$\begin{aligned}\bar{u}(t) = I_d\ddot{\theta}(t) & \\ &+ \int_0^1 \varrho(x)(x+r)\big(\ddot{w}(x,t) + (x+r)\ddot{\theta}(t)\big)dx \\ &- \int_0^1 R(x)\big(\ddot{\xi}(x,t) - \ddot{\theta}(t)\big)dx.\end{aligned}$$

We will define and demonstrate solutions of various problems connected with controllability and stabilizability of the introduced model.

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