Stabilization of a nonlinear system with elastic plates

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Consider a mechanical system that consists of a rigid body and two elastic plates. The vibration of the plates is governed by the Kirchhoff equations (see, e.g., [1]):

$$\ddot{w}_1 + a_1^2 \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right)^2 w_1 = (x_1 + d_1)\dot{\omega}_2 - (x_2 + d_2)\dot{\omega}_1, \tag{1}$$

$$\ddot{w}_2 + a_2^2 \left(\frac{\partial^2}{\partial x_1'^2} + \frac{\partial^2}{\partial x_2'^2} \right)^2 w_2 = (x_1' + d_1')\dot{\omega}_2 - (x_2' + d_2')\dot{\omega}_1, \tag{2}$$

subject to the boundary conditions

$$w_j|_{\partial\Omega_j} = 0, \qquad \left. \frac{\partial^2 w_j}{\partial n^2} \right|_{\partial\Omega_j} = 0, \qquad j = 1, 2. \tag{3}$$

We exploit the angular momentum equations for the rigid body-carrier:

$$\dot{K} + \omega \times K = f, \tag{4}$$

where $K = I\omega + \rho_1 \int\limits_{\Omega_1} r_P \times v_P dx + \rho_2 \int\limits_{\Omega_2} r_K \times v_K dx'$, and use the Poisson kinematic equations:

$$\dot{g}_{i1} = \omega_3 g_{i2} - \omega_2 g_{i3}, \; \dot{g}_{i2} = \omega_1 g_{i3} - \omega_3 g_{i1}, \; \dot{g}_{i3} = \omega_2 g_{i1} - \omega_1 g_{i2}, \quad i = \overline{1,3}. \tag{5}$$

We rewrite the control system (1)–(5) as an abstract differential equation with respect to the state ξ in a suitable Hilbert space H and propose a feedback law $f = G\xi$, so that the closed-loop system takes the form

$$\frac{d}{dt}\xi(t) = F\xi(t), \qquad F = A + BG. \tag{6}$$

Here $A : D(A) \to H$ is unbounded nonlinear operator, $B : \mathbb{R}^3 \to H$ is bounded linear operator, and $G : H \to \mathbb{R}^3$.

We prove that the solution $\xi = 0$ of equation (6) is strongly stable in the sense of Lyapunov.

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[1] Zuyev A., Novikova J. Modeling and stabilization of a rotating mechanical system with elastic plates // IFAC-PapersOnLine. - 2018. - Vol.51, No. 2. - P. 493-498.